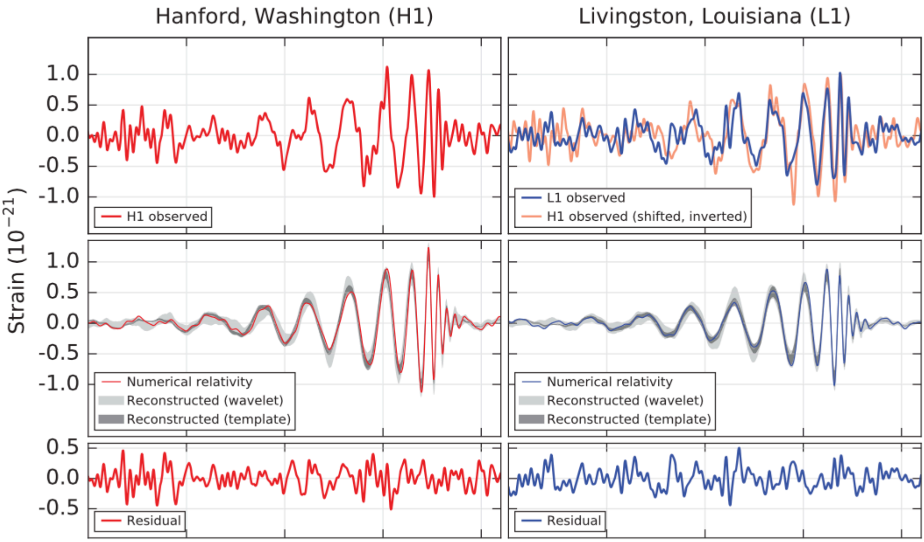
IOP Institute of Physics

Gravitational Physics Group

NEWSLETTER **September 2016**

Issue no. 11



First observation of Gravitational Waves from the LIGO experiment..

Figure courtesy: B.P. Abbott et al., PRL 116, 061102 (2016)

See http://gp.iop.org for more details

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Newsletter edited by Adam Christopherson. Please send comments or suggestions/nominations for future contributed articles to:

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Welcome from the Chair

Dear Members,

Welcome to another edition of our newsletter. On 28 and 29 Nov 2015, the Gravitational Physics Group held a meeting in Queen Mary University of London to celebrate the centenary of general relativity. The event included talks on scientific and historical aspects of the theory as well its influence on society. The two-day event was attended by more than 500 people with outreach talks for School children in the evening on the first day. The talks can all be found online at: http://astro.qmul.ac.uk/conferences/einsteins-legacy/celebrating-100-years-general-relativity.

Less than a year after proposing general relativity Einstein worked out wave-like solutions of his new theory but he and many others had difficulty with the notion of gravitational waves. Immediately after the Chapel Hill conference in 1959, Joseph Weber built a bar detector to observe gravitational waves when much of the theory community still doubted their existence. This conference is believed to be the starting point of what eventually laid to rest any concerns about the reality of gravitational waves. The discovery of the Hulse-Taylor binary in 1974 and the subsequent confirmation of gravitational radiation back-reaction in this system provided further impetus to experimental efforts to detect gravitational waves. Rai Weiss wrote in 1972 the first detailed design of a laser interferometer enumerating all the noise sources that must be overcome; this eventually culminated in the realisation of the Laser Interferometer Gravitational-Wave Observatory (LIGO) in the US.

On September 14, 2015, 100 years after GR and 5 decades after the first experiments of Weber, the twin LIGO instruments at Hanford, Washington and Livingston, Louisiana made the first direct detection of gravitational waves. The signal is consistent with a pair of black holes of about 36 and 29 solar masses. The inspiral and merger process resulted in a remnant black hole that is estimated to be roughly 63 solar masses, 3 solar masses being converted to pure ripples of spacetime geometry.

Gravitational waves interact weakly with matter and don’t suffer dispersion and scattering that so easily degrade electromagnetic waves as they propagate through the Universe. With LIGO, and other gravitational wave detectors (Virgo, KAGRA and LIGO-India) we will be able to explore the Universe in a radically different way.

The success of ground-based observations makes the case for Laser Interferometer Space Antenna (LISA) very compelling. With LISA we will be able to test the strong field dynamics of GR with exquisite precision and discover the growth of black holes throughout cosmic history. The LISA Pathfinder mission was launched last year and has been taking data to test key technologies that are needed for the success of LISA. By the time the newsletter goes to print we expect to hear that LISA technology is ready.

It is quite phenomenal that human endeavour to understand nature has produced instruments that are capable of detecting the miniscule effect that cosmic gravitational waves cause on test masses. What LIGO discovery has shown us today is that technology is only limited by fundamental physics principles; we can keep pushing the boundaries to achieve remarkable feats. This is a mammoth achievement for not only science but also for engineering, computing, material science, etc. I also hope that this will be inspirational for students to pursue careers in science.

The progress in science is slow but rewards, when they do come, are unmeasurable. It is not just an intellectual exercise but it is an adventure and the desire to go where no one has gone before and a new journey has just begun.

Best regards

B.S. Sathyaprakash

Events

Young Theorists' Forum 2014

Helen Baron

In December 2014 the seventh Young Theorists' Forum (YTF) took place at Durham University, an event which shows no sign of losing its well-earned popularity in the UK community of PhD students in theoretical physics. The two-day event ran in Durham as a follow-on to the Annual Theory Meeting with the purpose of giving PhD researchers a platform to present and discuss their work with their peers in an environment conducive to the fostering of inter-institutional collaboration.

Short (15 or 30 minute) talks were given by 37 of the 91 student participants in the following sessions: BSM Physics, String Phenomenology & Holography, Cosmology, Amplitudes & Solitons, Phenomenology, String Theory, QCD, Gravity. The sessions were punctuated by coffee breaks allowing the participants, from 17 different universities, ample opportunity to discuss the talks further.

The highlight, at the heart of the forum, was the plenary talk by Professor Gerald Dunne from University of Connecticut entitled "Resurgence and the Physics of Divergence". The talk was generally perceived as extraordinarily clear and very inspiring. It was followed by a relaxed pizza evening where participants had time for many more interesting discussions. Posters were presented during and after the pizza dinner, thus giving even more students the opportunity to present and discuss their research.

For more details on the event please check out the conference website: http://maths.dur.ac.uk/YTF/2014/home.html Due to the great success of the event this year and in previous years, we hope to be able to host it again in December 2015.

The organisers are grateful for the funding received by the Mathematical and Theoretical Physics, High Energy Particle Physics and the Gravitational Physics groups of the IOP and for funding received from the Scottish Universities Physics Alliance (SUPA) and from the Centre for Academic and Researcher Development at Durham University. Our gratitude also goes to Durham University.

BritGrav 2015

Christopher Berry

The 15th British Gravity (BritGrav) Meeting was held on 20–21 April 2015 at the University of Birmingham. The meeting had 81 registered participants and 50 talks across two days! The meeting covered a wide range of topics, with talks on everything from gravitational lensing to black-hole atoms.

The BritGrav Meetings are primarily targeted at engaging and showcasing the work of young researchers. BritGrav 15 was particularly successful in this regards with 69% of participants being students and 78% of talks being given by students. We were lucky to have two talks from recipients of the Gravitational Physics Group's thesis prize: Anna Heffernan entertained us with details of her work on the gravitational self-force, and Patricia Schmidt explained her successes in modelling the precession of black-hole binaries.

There was a prize for best student talks, sponsored by Classical & Quantum Gravity. The runners up were Viraj Sanghai for his talk on post-Newtonian cosmologies, and Umberto Lupo for his discussion of black holes in a box. The winner was Christopher Moore for his talk on including model uncertainty into gravitational-wave parameter estimation. We thank our panel of judges (John Miller, Carsten Gundlach and John Veitch) for their careful deliberations.

Outside of talks, Monday evening included a wine reception courtesy of Classical & Quantum Gravity, followed by the conference dinner and much lively discussion. On Tuesday evening, we held a public lecture. Jim Hough gave a talk on the advances towards detection of gravitational waves. This was extremely well received and was attended by over 100 members of the public.

This meeting was kindly sponsored by the Gravitational Physics Group, and the journal Classical & Quantum Gravity. The IOP Gravitational Physics Group was particularly generous in providing funds to support students attending the meeting, increasing their original allocation following high demand. I am also extremely grateful for everyone in the University of Birmingham's Gravitational Physics Group who helped to organise and run the meeting so smoothly.

Highlights from BritGrav 15 can be found at http://storify.com/cplberry/britgrav-15

Einstein's Legacy: Celebrating 100 Years of General Relativity

Tim Clifton

The year 2015 saw the one hundred year anniversary of Einstein's theory of General Relativity. Of course, the development of the theory itself wasn't completed in a single year. It took Einstein a decade of effort to go from his special theory to the general one, and there were a number of milestones along the way (as well as a few hiccups). The final breakthrough, however, occurred on the 25th of November 1915, when Einstein presented his field equations to the Prussian Academy of Sciences. This was the point when the world became aware of his revolutionary new theory.

The international community, coordinated by the International Society on General Relativity and Gravitation, decided to organise meetings around the world, in order to celebrate this occasion. The organisation of the UK meeting fell to the Gravitational Physics Group of the Institute of Physics, and we decided to hold our meeting on the weekend of the 28th and 29th of November, at Queen Mary University of London. This was the closest weekend to the anniversary of the presentation itself, and Queen Mary seemed like the perfect venue. It was here that figureheads like George McVittie and Bill Bonnor spent the majority of their careers, and where relativity is still an active topic of research in both the maths and physics departments.

It was decided that the meeting should try and cover as many aspects of GR as possible, so we invited speakers from variety of different disciplines from the maths and physics communities, as well as from the humanities. We decided to have public talks on the evening of the 28th, and to include an outreach event on the same day in order to try and engage as many school students as possible. The idea was to have scientific talks running in parallel to outreach events, and to bring everyone together for the public lectures on Saturday.

The event kicked off with an opening address from Malcolm MacCallum, who has been a professor and Queen Mary for many years, as well as being a leading international figure in the mathematical relativity community. This was followed by a historical introduction to the development of General Relativity by Andrew Robinson, and a lecture by Richard Staley on the philosophical ideas that Einstein was wrestling with when he developed the theory. These two speakers are experts in both the man himself, and the ideas that went into his theory. They gave an excellent account of those very early days of relativity.

The afternoon talks of the first day then focussed on some of the scientific fields that emerged from GR. We had talks from Bangalore Sathyaprakash, Jim Hough and Pedro Ferreira on gravitational waves, experimental gravity, and cosmology. These three figures are all leaders in their respective fields, and gave an excellent series of talks. This was followed by a poster session, and an outreach event that ran in parallel to the talks in a different part of the university. The outreach events were organised by volunteers from Southampton, Glasgow and Queen Mary Universities, and was coordinated by Nils Andersson.

A volunteer engaging with the public during the outreach event.

The school students, and their teachers, took part in a total of 12 different outreach activities. These included games, discussions, and demonstrations of a wide variety of different effects and phenomena related to GR, as well as a taster lecture from Nils to introduce them to the ideas. We had interferometers, wave machines, computer games, posters and simulations, as well as snacks and drinks, for the students and teachers to enjoy. After this they were taken to the venue for the main meeting, where John Barrow and Sir Roger Penrose gave public talks on the consequences of GR for our understanding of the Universe.

Sir Roger Penrose delivering a public talk during the meeting.

The second day of the meeting was led by Queen Mary's own Katy Price, who spoke about the reception of Einstein's ideas in the popular press at the time. We then had physics and maths talks from Ramesh Narayan, Mihalis Dafermos and Mike Duff. These three leading lights gave expert descriptions of black holes, mathematical relativity and quantum gravity. The last talk of the meeting was then given by Harry Collins, who spoke about the sociology of the study of relativity, before the event was finally wrapped up.

In total, around 500 people attended the meeting, including about one hundred and fifty 16-18 year olds. The feedback I received from everyone involved was all very positive and constructive, but the event itself couldn't have happened without a large number of people giving up a lot of their time. This included all of the speakers (of course), as well as the Gravitational Physics Group committee members, the large number of helpers who designed and ran the outreach event, the Events Team and academic staff at Queen Mary, and our sponsors who provided generous financial support (these include the IOP, the RAS, Cardiff University, Queen Mary University of London, the University of Nottingham, and the LMS).

Legacy pages for the meeting can be found at http://astro.qmul.ac.uk/einstein/

A view of the audience engrossed in the talks

Prizes

2015 GPG Thesis Prize

The IOP's Gravitational Physics Group's thesis prize for 2014, co-sponsored by *Classical and Quantum Gravity*, has been awarded to Dr Patricia Schmidt. Dr Schmidt's thesis constitutes a substantial contribution to the study of gravitational wave emission from inspiralling compact binary systems. The judging panel was very impressed, not only with the scientific content, but also with the excellent presentation on the importance of precession in black hole binaries and its relevance to gravitational astronomy.

Dr Schmidt obtained her PhD from Cardiff University before taking up a postdoc at Caltech in 2015. You can also read an interview with Dr Schmidt on CQG+.

2016 GPG Thesis Prize

The IOP's Gravitational Physics Group's thesis prize for 2014, co-sponsored by *Classical and Quantum Gravity*, has been awarded to Dr John Muddle, of the University of Southampton. His thesis was "Advanced Numerical Methods for Neutron Star Interfaces", and received the following commendation from the judges:

"Dr Muddle's thesis was beautifully presented, with physical descriptions and mathematical explanations that were well balanced and highly complementary. The thesis provided an excellent overview of the subject of neutron star interfaces, and was both engaging and enlightening. This is an exceptional piece of work, and although the competition was very tough, Dr Muddle's thesis was unanimously agreed by all the judges to be this year's winning entry".

Contributed Articles

Using Gaussian processes to overcome model errors

Christopher Moore, Institute of Astronomy, University of Cambridge
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Introduction

The advanced LIGO detectors have recently made the first direct detection of gravitational waves (GWs). These waves were produced by the inspiral and coalescence of two black holes. LIGO also aims to detect the GWs from the coalescence of two neutron stars, or a black hole and a neutron star. For the case of a binary BH (BBH), the high mass of the system compared to a binary neutron star means that the frequency of GWs emitted around the merger lies near the centre of the sensitivity band of the ground-based detectors; around a few hundred Hertz. The two advanced LIGO (aLIGO) interferometers [1] conducted their first operating run between September 2015 and January 2016. Advanced LIGO will soon be joined by the advanced VIRGO detector [2], the cryogenic KAGRA detector [3], and another LIGO detector in India [4]; these will operate in as a network. The event rate for BBH mergers in the universe is still highly uncertain; rate estimates based on the first release of advance LIGO data lie in the range 2-400 $Gpc^{-3}yr^{-1}$ [5].

Both the detection and the subsequent characterisation of a binary system will rely heavily on detailed models (or templates) of the emitted GW signal. Detection of such sources is most efficient when the model signal matches faithfully with the true physical signal. For parameter estimation the faithfulness of the model is even more important; inaccurate models lead to biased estimates of the system parameters, including the component objects masses and their spins. The requirement for accurate models is especially difficult to meet for BBH sources as these sources undergo inspiral (the gradual shrinking of the orbital separation due to the emission of GWs), merger (below a certain separation stable orbits are no longer possible and the two objects plunge together), and ringdown (the system settles down rapidly into its final state), all in the sensitivity band of the detector. Therefore, the analysis of BBH signals requires models of the signal which remain accurate throughout all of the inspiral, merger, and ringdown (IMR) phases of the signal. Additionally, the models will ideally include all the relevant physics: for example, precessing BH spins, eccentricity of the orbit, high order modes of gravitational radiation, and so on. While numerical relativity (NR) simulations that meet the above requirements have recently become available, the high computational cost has, until now, prevented their use for parameter estimation.

Instead, NR simulations have been used to calibrate a range of computationally cheaper semi-analytic IMR *approximants*, such as the effective-one-body–NR (EOBNR) [6] and IMRPhenom [7] models.

The use of imperfect models for parameter estimation is a known problem for both ground-based [8] as well as space-based [9] detectors. This parameter estimation problem occurs because the model signal which best matches the physical signal present in the data has source parameters which are in general offset from the true physical values; this offset leads to systematic error in the recovered parameters. As noted in [9] this systematic error is independent of the signal-to-noise ratio (SNR) with which the source is observed. Because the random statistical errors due to the presence of noise in the data scale inversely with the SNR, above some critical value the parameter estimation error will always be dominated by the systematic model error.

Recently a novel approach for dealing with the parameter estimation problem was proposed. The unknown error which is inevitably present in the signal model is treated as nuisance parameter in a Bayesian analysis. This nuisance parameter is then marginalised over using the technique of Gaussian process regression (GPR) to interpolate a training set of accurate waveforms which are generated “offline” (i.e. the training set can be generated before the data is taken, and before any parameter estimation study is started). In this way the computational cost of the “online” parameter estimation is largely unaffected. The likelihood obtained from this marginalisation process is called the marginalised likelihood [10,11].

In this article the marginalised likelihood, as it was originally proposed, is briefly reviewed and then generalised in such a way as to allow for frequency dependent model errors. Frequency dependent errors are important in the case of BBH signals as the signal models typically have much smaller errors in the low frequency inspiral than in the higher frequency merger. It is then shown how the frequency dependent model error can be interpreted as an effective source of noise in the experiment. This is then illustrated in a toy problem by calculating the effective noise that would be present if a BBH signal (containing inspiral, merger and ringdown) were analysed using an inspiral only model.

The marginalised likelihood

The gravitational wave signal, $h(\lambda)$, is given as a function of the vector of source parameters, λ, which include quantities such as the masses and spins of the two objects. The time-series of measured data, s, from a GW detector consists of noise, n, and possibly a signal; i.e. $s = n$ or $s = h(\lambda) + n$. After initial detection of the GWs has indicated the presence of a signal in a given segment of the data the aim of a subsequent parameter estimation study is to infer the source parameters from the measured data. We seek the posterior probability distribution on the parameters, $P(\lambda|s)$. This is given by Bayes' theorem,

$$P(\lambda|s) = P(s|\lambda)\frac{P(\lambda)}{Z(s)}, \quad (1)$$

in terms of the prior distribution on the source parameters, $P(\lambda)$, the normalising evidence, $Z(s)$, and most importantly the likelihood,

$$P(s|\lambda) \propto \exp(-\langle s - h(\lambda)|s - h(\lambda)\rangle)\,. \quad (2)$$

In a typical situation of interest the likelihood is given in terms of the exponential of a signal inner product of the difference between the measured detector data and the model signal. The signal inner product is defined as

$$\langle a|b\rangle = 4\Re\left[\int_{-\infty}^{\infty} \mathrm{d}f\ \frac{\tilde{a}(f)\tilde{b}(f)^*}{S(f)}\right], \quad (3)$$

where the detector noise is assumed to be stationary and Gaussian with noise power spectral density (PSD) $S(f)$.

However, the true gravitational wave signal, $h(\lambda)$, is not usually available, at least not at a reasonable computational cost. Instead, what is available is the approximate signal model, $H(\lambda)$, which is related to the true signal by some unknown model error, $\delta h(\lambda)$,

$$\delta h(\lambda) = H(\lambda) - h(\lambda)\,. \quad (4)$$

If the approximate model was simply used in place of the accurate model in the standard expression for the likelihood an approximation to the likelihood in Eq 2 would be obtained. This would suffer from the detection and parameter estimation problems discussed above.

To overcome these problems with the approximate likelihood the unknown model error, $\delta h(\lambda)$, is treated in the same manner than an unknown nuisance parameter would usually be treated in a Bayesian analysis: it is marginalised over. We propose to use the following marginalised likelihood in place of the standard likelihood in Eq 2,

$$\mathcal{L}(s|\lambda) = \int \mathrm{d}(\delta h)\ \ P(\delta h)\times \exp(-\langle s - h(\lambda) + \delta h(\lambda)|s - h(\lambda) + \delta h(\lambda)\rangle)\,. \quad (5)$$

In order to evaluate the marginalised likelihood it is necessary to (i) specify the prior, $P(\delta h)$, on the model error, and (ii) evaluate the integral in Eq 5.

Firstly, to construct the prior, assume that it is possible to compute highly (but not necessarily perfectly) accurate model signals at the offline stage of the analysis where computational constraints are less limiting. In practice this might be done using NR simulations, computationally expensive time-domain models which include a wide range of physical effects, or (more likely) a combination of all of the best available methods. Using the results of these offline calculations a training set, $\mathfrak{D}$, is constructed containing *n* model errors spread over the region of parameter space of interest,

$$\mathfrak{D} = \{(\delta h(\lambda_i), \lambda_i);\ i = 1,2,\cdots,n\}\,. \tag{6}$$

To interpolate the training set to obtain the model error at some new point, λ, in parameter space we use the technique of GPR [12,13]. This process assumes that the model errors in the training set are correlated across parameter space according to a known covariance function, *k(λ,λ')*. The functional form of *k* must be specified, but can include some unknown parameters (known as hyper-parameters) which can be "learnt" from the properties of the training set. The GPR interpolant provides a Gaussian distribution for the model error at some arbitrary point in parameter space, this forms the prior on the model error needed in Eq 5. This distribution is given by

$$\delta h(\lambda) \sim \mathcal{N}(\,\mu(\lambda),\ \sigma^2(\lambda)\,)\ \text{, where} \tag{7}$$

$$\begin{aligned}\mu(\lambda) &= \sum_i k(\lambda,\lambda_i)\,\delta h(\lambda_i)\ \text{ and }\ \sigma^2(\lambda)\\ &= k(\lambda,\lambda) - \sum_{ij} k(\lambda,\lambda_i)\,k(\lambda,\lambda_j)\,\mathrm{inv}\big[k(\lambda_i,\lambda_j)\big]\,.\end{aligned} \tag{8}$$

Secondly, to evaluate the integral in Eq 5, since the both exponential terms in the integrand are now Gaussians in $\delta h(\lambda)$ the entire integrand is a Gaussian and may be simply evaluated analytically to give the following expression for the marginalised likelihood,

$$\mathcal{L}(s|\lambda) = \frac{1}{\sqrt{1+\sigma^2(\lambda)}}\exp\left(-\frac{\langle s - H(\lambda) + \mu(\lambda) | s - H(\lambda) + \mu(\lambda)\rangle}{1+\sigma^2(\lambda)}\right). \tag{9}$$

The expression for the marginalised likelihood in Eq 9 was first derived in [10]. This marginalised is given in terms of the cheaper, approximate waveform model,

$H(\lambda)$, but with corrections entering from the training set via the GPR. The GPR quantity $\mu(\lambda)$ is the interpolation of the model error across parameter space, this acts to shift the peak of the marginalised likelihood relative to the peak of the standard likelihood, hopefully reducing the systematic error in the parameter estimates. The GPR quantity $\sigma^2(\lambda; f)$ is the uncertainty on the interpolation, this appears in the denominator inside the exponential. The GPR uncertainty acts to broaden any peaks in the likelihood thereby increasing the uncertainty in the parameter estimates to reflect the uncertainty in the model. The relative importance of these two effects, shifting and broadening the likelihood peaks, varies depending on where the true signal happens to lie relative to the points in the training set. The GPR uncertainty also appears in the normalisation outside of the exponential, however this term typically varies much more slowly with λ than the exponential term does and hence is usually negligible for parameter estimation purposes.

The marginalised likelihood provides a negligibly more complicated expression for the likelihood which accounts for the imperfections in our model waveforms in a fully Bayesian way. It has also been shown that this marginalised likelihood reduces, and in some cases completely removes, the systematic bias in the parameter estimates obtained [14]. The marginalised likelihood also provides a way for additional physics (e.g. spin precession, eccentricity, higher modes, etc) to be included in the simple cheap model $H(\lambda)$. For example if the accurate model includes the effects of eccentricity, $h(\lambda, e)$, then the training set may include waveform differences for eccentric signals, $\delta h(\lambda, e) = H(\lambda) - h(\lambda, e)$. When this training set is interpolated across the extended parameter space (λ, e) the resulting cheap interpolated waveform, $\mu(\lambda, e)$, inherits some of the effects of eccentricity from the training set.

Frequency dependent model errors

One criticism of the above approach is the overly simplistic way in which the frequency behaviour of the model error was treated. In particular when carrying out the GPR interpolation a covariance of the form *k(λ,λ’)* was assumed; i.e. the waveform error is correlated only in parameter space, and is uncorrelated in frequency. In one sense this was a good choice, assuming a lack of correlation is a conservative choice in the sense that it leads to larger values of the GPR uncertainty, $\sigma^2(\lambda)$, and a broader and more conservative posterior. On the other hand, properly accounting for the frequency dependence of $\delta h(\lambda; f)$ will yield a better estimate of the true likelihood surface.

To achieve this generalisation we take in place of the training set in Eq 5, the model errors evaluated at each frequency bin,

$$\mathfrak{D}_j = \{(\delta h(\lambda_i; f_j), \lambda_i);\ i = 1,2,\cdots,n\}, \qquad \text{where } j = 1,2,\cdots,m. \tag{10}$$

This gives m different training set each of which will be separately interpolated. The model errors are evaluated in the frequency domain, and have m frequency bin separated by $\delta f = 1/T$, where T is the total time length of the signal. A more general approach would combine all of these training sets into one set of size $m \times n$, this would allow the GPR to account for the correlations between different frequency bins. However, here we choose to take this more restrictive approach*),* this is an intermediate case between what was done in [10,11] and the fully general case. As was the case in [10,11] the justification for restriction in the covariance function comes from the performance of the resultant marginalised likelihood.

We may now interpolate the new training set, again using the technique of GPR, but this time using the new covariance function, ${\sigma_f}^2(f_j)k(\lambda, \lambda')$*.* The overall scale of the model error at each frequency is governed by $\sigma_f^2(f_j)$ and this is may be "learnt" by the GPR from the training set in the same manner as any other hyper-parameter. Interpolating this set combination of training sets gives a new expression for the prior distribution of the waveform error. As before this is a Gaussian distribution and so the marginalisation integral in Eq 5 can be carried out analytically. For the sake of brevity we do not present details of this process here, but the resulting expression for the marginalised likelihood is

$$\Lambda(s|\lambda) = \prod_j \left\{ \frac{S(f_j)}{S(f_j) + 4\,\delta f\,\sigma^2(\lambda; f_j)} \exp\left(-2\,\delta f\,\frac{|\tilde{s}(f_j) - \widetilde{H}(\lambda; f_j) + \tilde{\mu}(\lambda; f_j)|^2}{S(f_j) + 4\,\delta f\,\sigma^2(\lambda; f_j)} \right) \right\}. \tag{11}$$

The new expression for the marginalised likelihood in Eq 11 retains many of the advantages of the original expression in Eq 9, however, now the GPR uncertainty term which controls the broadening of the likelihood is a function of a frequency, $\sigma^2(\lambda; f)$.

The model error viewed as a source of noise

The new expression in Eq 11 for the marginalised likelihood may be rewritten in a suggestive way,

$$\Lambda(s|\lambda) \propto \prod_j \left\{ \frac{S(f_j)}{S'(\lambda; f_j)} \right\} \exp(-[s - H(\lambda) + \mu(\lambda)|s - H(\lambda) + \mu(\lambda)]) \,. \tag{12}$$

Here the square brackets denote a new inner product on the signals defined as,

$$[a|b] = 4\Re\left(\int_{-\infty}^{\infty} \mathrm{d}f \; \frac{\tilde{a}(f)\tilde{b}(f)^*}{S'(f)}\right). \tag{13}$$

The new signal inner product is identical to the original signal inner product but with the noise PSD replaced by a new effective noise PSD, $S(f) \rightarrow S'(f) = S(f) + 4\sigma^2(\lambda; f)/T$.

The new expression for the marginalised likelihood looks almost identical to the standard likelihood in Eq 2 except with a modified inner product. This motivates the interpretation of $4\sigma^2(\lambda; f)/T$ as a source of noise in the experiment. Other sources of noise include, for example, seismic noise, thermal noise, shot noise, etc. The noise PSD for each of these processes combine to give the detector noise PSD, $S(f) = S_{\text{seismic}}(f) + S_{\text{seismic}}(f) + S_{\text{seismic}}(f) + \cdots$. These sources of noise arise from physical processes which act to shake the experiment thereby obscuring the presence of any GW signal. In contrast $4\sigma^2(\lambda; f)/T$ is purely a theoretical property of our models (in particular it quantifies the disagreement between the approximate and accurate signal models used). Nevertheless, despite their very different origins $S(f)$ and $4\sigma^2(\lambda; f)/T$ enter the expression for the marginalised likelihood in Eq 12 in (almost) the same way (the slight difference arises from the presence of $4\sigma^2(\lambda; f)/T$ in the normalising pre-factor, however as argued above this is typically negligible compared to the exponential term for parameter estimation purposes).

Our ability to extract accurate parameter estimates from GW signals in limited both by the presence of noise in the experiment, and by the accuracy of the models we choose to use for the analysis.

A comment is needed on the presence of the total time length of the data, T, in the model error contribution to the effective noise PSD. It may seem strange that the effective noise should depend on the length of data that was chosen for the analysis. In particular, the model error contribution to the effective noise can be made arbitrarily small in comparison to the detector noise contribution by analysis a sufficiently long segment of data. This arises from the fact that $\langle \delta h | \delta h \rangle$ is a signal power, whilst $S(f)$ is a power spectral *density.* For a fixed duration signal, δh, as T is taken arbitrarily long the uncertainty in the total signal is dominated by the physical noise processes rather than the fixed signal. Normalising the model error by dividing by the signal length produces a PSD like quantity which can be compared with $S(f)$.

The model error contribution to the noise may be plotted alongside the detector contribution and their relative significance compared. In order to illustrate this here, the situation were a BBH signal is analysed using an inspiral only model is considered. For the accurate waveform model, $h(\lambda)$, an inspiral, merger and ringdown model known as IMRPhenomC [15]; this models the entire signal using a single effective spin parameter (roughly the total spin aligned with the orbital angular momentum) to describe the spins of both black holes. For the approximate model, $H(\lambda)$, an inspiral only model known as TaylorF2 model [16-18] was chosen; this is a fast frequency domain model for the inspiral only part of the waveform which accounts for the component of each BH spin aligned with the orbital angular momentum. This choice of models was not intended to be representative of a real data analysis situation, in particular one would never choose to analyse a BBH signal with an inspiral only waveform model. Instead here our aim was to illustrate how the model error, which here is the absence of the merger and ringdown parts of the signal from $H(\lambda)$, can be viewed as an effective source of noise in the experiment.

For simplicity the case of an equal mass, non-spinning binary, was considered. Three different masses were considered, the component masses were $3M_{\odot}$, $5M_{\odot}$, and $10M_{\odot}$. The binary system was simulated as being observed face-on with a single interferometer which has the noise power spectral density shown by the solid line in Fig 1. The distance to the source was fixed in each case by requiring that the observed signal has an optimum SNR of 12. In each case the difference between the accurate and approximate models, $\delta h(\lambda; f)$, was calculated. The GPR uncertainty depends not only on the model error, but also on the positioning of the training set points. Here, for simplicity, the worst case scenario was considered where the true signal occurs in a region of parameter space with no training set points and hence $\sigma_f^2(\lambda; f) = \langle \delta h(\lambda; f) | \delta h(\lambda; f) \rangle$. This was used to calculate the model error contribution to the total effective noise PSD, $4\sigma^2(\lambda; f)/T$. The length of the signal, T, was fixed in each case as the next power of 2 in seconds above the estimated length of the signal starting from a lower frequency of $10\ \mathrm{Hz}$, this is representative of a choice that might be made in a real data analysis situation. The results are plotted in Fig 1; in each case the total effective noise may be obtained by adding the detector PSD and the model contribution, $S'(f) = S(f) + 4\sigma^2(\lambda; f)/T$.

For the low mass ($m_1 = m_2 = 3M_{\odot}$) BBH, where the observed part of the signal is dominated by the inspiral and the merger and ringdown occurs at higher frequencies, the detector noise is larger than the model error noise at all frequencies. In this case it can also be seen from Fig 1 that the approximate model remains in phase with the accurate model for a significant fraction of the signal as the model error undergoes of order one oscillation across the bandwidth of the detector. In this situation the marginalised likelihood would recover the standard expression for the likelihood to high accuracy. For the high mass ($m_1 = m_2 = 10M_\odot$) BBH, where the merger and ringdown contribute significantly to the observed SNR, the model error noise is larger than the detector noise across most of the detector bandwidth. The model error contribution to the noise oscillates across the bandwidth of the detector due to the approximate model drifting into and out of phase with the accurate model. In this situation the marginalised likelihood would offer a significant improvement over the standard likelihood.

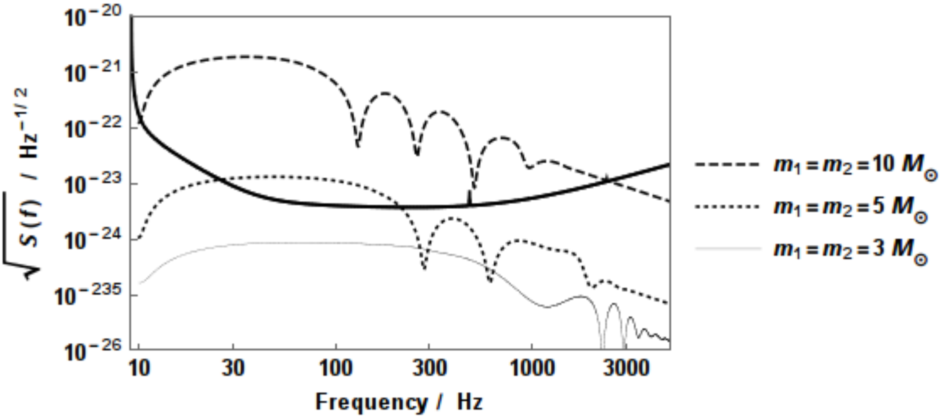

Figure 1: *The dashed, dotted and thin lines show the model error contribution to the effective noise in the marginalised likelihood in Eq 12. The model error noise is most significant for the higher mass systems where the merger contributes most to the SNR. The solid black curve shows the (square root of) the expected noise power spectral density of aLIGO at design sensitivity in the zero-detuned high-power configuration [19].*

Summary

Gravitational wave astronomy relies heavily on detailed models of the source, this is especially true for binary sources where these models may be produced with a reasonable degree of certainty. However, some model independent approaches for detection and waveform extraction do exists [20-22], and for high mass black hole ($m_1 + m_2 \approx 100M_\odot$) the sensitivities of these approaches can approach that of the optimal matched filter.

Inaccuracies in the models are unavoidable even in the best available NR simulations. The best available NR waveforms are starting to become available in larger numbers, although these are still typically fairly short waveforms (typically a few tens of orbits) and are definitely not available in sufficient numbers to be used alone in parameter estimation studies. This forces the community to make use of computationally faster approximate models, usually computed in the frequency domain. Model errors are a known problem for both ground- and space-based detectors.

The marginalised likelihood has recently been proposed to tackle some of the problems associated with model uncertainties. As originally proposed the marginalised likelihood placed some very restrictive assumptions on the frequency behaviour of the model error; these assumptions were not necessary, they were made to simplify the resultant expression for likelihood. Here some of those assumptions have been relaxed and a new expression for the marginalised likelihood obtained. In particular, the uncertainty in the interpolation of the model error is now allowed to vary with frequency.

We have shown how the new expression for the marginalised likelihood may be written in a form that looks very much like the standard likelihood used in GW parameter estimation, except the noise PSD in the inner product now has an additional contribution arising from the model error. This allows us to identify a particular function of the model uncertainty as an additional source of noise in the experiment. Our ability to extract useful science from a gravitational wave detector is constrained by both the physical noise in the instrument (including, thermal, seismic shot noise etc.) and by the effective noise caused by our inaccurate models. This was illustrated by considering the situation were a BBH signal is analysed with an inspiral only signal model.

Acknowledgements

This work would not have been possible without the help of Jonathan Gair, Christopher Berry, and Alvin Chua.

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