

The timing residual (difference between the measured times of arrival of pulses and the timing model) for the crab pulsar, collected using data from the Crab ephemeris.

Figure courtesy G. Ashton

Table of Contents

Welcome from the Chair	3
Events.....	5
Young Theorists' Forum 2013	5
New Frontiers in Dynamical Gravity	5
BritGrav 2014	6
Prizes.....	8
2014 GPG Thesis Prize	8
Contributed Articles	9
The effect of timing noise on continuous gravitational wave searches	9
Black holes and scalar hair	17
Untangling precession: Modelling the gravitational wave signal from precessing black hole binaries	25
Group Committee	33
Information about the group and membership.....	33

Newsletter edited by Adam Christopherson. Please send comments or suggestions/nominations for future contributed articles to:

achristopherson@gmail.com.

Welcome from the Chair

Dear Colleagues,

Happy 2015 and welcome to another edition of the Gravitational Physics newsletter. At the outset I would like to thank the new committee members Davide Gerosa, Cambridge, and Silke Weinfurter, Nottingham, for agreeing to serve on the committee; they are our link to younger (not necessarily in age) members of our field and I very much value their input. I would also like to thank all colleagues on the committee for their insight and inputs on our programme.

This is the centennial year of General Relativity. ISGRG has declared November 25th as the Centennial Day - the day Einstein submitted his paper to the Prussian Academy. A number of meetings are being held around the world in celebration of Einstein's legacy and how the field has grown over the past century. To mark the occasion, the IOP Gravitational Physics is organising a two-day meeting at Queen Mary, London, on the 28 and 29 of November 2015. The meeting aims to explore Einstein's influence on art, philosophy and culture and, in particular, how British science was shaped by his work, starting from Eddington to Hawking and beyond. Please mark these dates in your diaries and I hope to see many of you at what we expect to be a fantastic gathering with lively lectures for academics and the public alike.

2015 also marks the beginning of a new era in experimental gravity. LIGO and Virgo are both being upgraded to achieve a thousand-fold increase in survey volume than what they had accomplished during their initial operations. The first science runs of upgraded detectors should start later this year and they are expected to reach design goals by 2018/19. A cryogenic detector called KAGRA is being built in the Kamioka mines in Japan, which will break new ground in detector technology, and we are awaiting the final decision on building an interferometer in India. The field has progressed remarkably over the past 20 years, achieving sensitivity levels undreamt even by Einstein.

This is a great time to be working on GR with many challenges in quantum gravity, to black hole firewall and detecting primordial gravitational waves to understanding dark energy. Many open questions remain but there are new opportunities to answer them from numerous astrophysical observations and physics experiments. Gravity will be tested in the coming years in ways never imagined before.

On the occasion of the centenary, it is tempting to speculate where the field will be in 100 years time, although, quoting Niels Bohr, prediction is difficult, especially about the future. Even so, over the next 100 years we can expect many of the current problems to have been solved including detecting gravitational waves from ground, space and underground and using this new tool to understand the Universe. GR may fail at some point and I think the next century will witness more concrete evidence of the need to once more modify our conception of spacetime, as hinted by certain quantum gravity theories. New theories of gravity, either classical or quantum, may be devoid of singularities, which would have consequences for both black holes and the Big Bang paradigm. Observations should also shed light on what black holes are and begin to provide some glimpse of what they might host - namely hints of quantum gravity.

One of the biggest debates of the current century, triggered in parts by quantum mechanics and cosmology, has to be the multiverse and anthropic principle. It troubles me that we are debating about things that, in principle, cannot be tested, even if some colleagues disagree that the multiverse is untestable. It is not the first time that physicists have come to face such a massive division but I believe what unifies us is our willingness to forego prejudice in the face of experimental evidence. I hope the next century will give us the tools to address this debate in a meaningful and scientific way.

This year's British Gravity meeting (BritGrav15) will be held at the University of Birmingham on April 21 and 22. Please encourage your students and postdocs to attend and present their work.

With that, let me wish you all a very happy centenary.

B.S. Sathyaprakash,
Chair IOP Gravitational Physics
Group Cardiff University, Cardiff, UK

Events

Young Theorists' Forum 2013

Brian Colquhoun

December saw the sixth edition of the Young Theorists' Forum (YTF), an event which has proven to remain popular among doctoral students in theoretical high energy physics from around the UK. The conference took the form of a two day meeting on the 18th and 19th of December following the Annual Theory Meeting and held in Durham University's Department of Mathematical Sciences.

Of the 81 students in attendance, 35 gave talks about their research in one of the following sessions: SUSY Gauge Theories and Amplitudes; BSM Physics; String Theory; Lattice Field Theory; Gauge Theories and Gravity; Phenomenology; Solitons and M-theory; and Cosmology. A plenary talk took place at the end of the first day, given by Professor Lionel J. Mason of the University of Oxford. Following this talk, there was a pizza evening, encouraging discussion between attendees in an informal setting.

The generous support of the sponsors meant that the accommodation expenses were covered for all participants whether they presented their work or not. It is the hope that the seventh edition of the YTF will be organized to coincide yet again with the Annual Theory Meeting in Durham at the end of 2014.

New Frontiers in Dynamical Gravity

Helvi Witek

From March 24th to 28th, 2014, we hosted the workshop "New Frontiers in Dynamical Gravity" or, in short, "Gauge/gravity duality meets Numerical Relativity meets fundamental maths", at the futuristic site of DAMTP at the University of Cambridge, organized by P. Figueras, H. Reall, U. Sperhake and H. Witek. In this workshop, we have brought together leading experts in these fields. The schedule of the conference – typically two, hour-long overview talks in the morning and four half-hour talks in the afternoon – left plenty of time for fruitful discussions, the exchange of ideas and starting of new collaborations. The slides, as well as the group photo of the conference, are available on the website: <http://www.ctc.cam.ac.uk/activities/adsggrav2014/>.

The gauge/gravity correspondence offers a powerful tool to explore strongly coupled conformal field theories in $D-1$ dimensions by investigating gravity in the asymptotically anti-de Sitter (AdS) spacetimes in D dimensions, and

vice versa. In this conference, we have seen a number of talks exploiting the fluid/gravity correspondence to mimic, e.g., collisions of heavy ions at the RHIC or LHC by modelling collisions of shock waves in AdS. Instead, investigating black holes in AdS with non-Killing horizons provides a unique playground to study plasma flows with a spatially varying temperature profile. Conversely, the gauge/gravity dictionary can be used to better understand quantum gravity effects near the black hole singularity by studying specific conformal field theories, or by characterizing extremal surfaces reaching from the boundary into the bulk. Another flavour of this duality, namely the correspondence between gravity and condensed matter physics, has been employed to investigate phase transitions between insulators, metals and superconductors which are still poorly understood in solid state physics. This can be accomplished by exploring gravity coupled to Maxwell and scalar fields with a periodic potential.

A number of presentations have focused on the recently discovered turbulent instability of AdS. Numerical simulations of critical phenomena uncovered that (scalar) perturbations in AdS generically lead to black hole formation, while highly fine-tuned setups can represent “islands of stability”.

Another series of talks was concerned with more fundamental mathematical issues, such as the linear and non-linear stability of black holes both in asymptotically flat and AdS spacetimes and the structure of the black hole singularity. Last but not least, we have seen presentations of black holes in higher dimensional spacetimes, including remarkable results of quasi-normal mode and stability calculations in the large D limit and revisited numerical simulations investigating the non-linear stability of five dimensional rotating black holes.

On Wednesday evening we gathered for a feast at the beautiful Trinity College. After indulging in excellent food and wine, H. Reall gave an inspiring speech recapturing the first days of our conference and recalling some anecdotes about Sir Isaac Newton. One of the highlights of this conference was the visit to the COSMOS supercomputer, which is part of the DIRAC HPC facility funded by STFC and BIS.

BritGrav 2014

Jonathan Gair

The fourteenth British Gravity meeting took place on March 31st and April 1st 2014 in St Catharine's College, Cambridge. Attendance at this meeting was very high, up more than fifty per cent on the year before, with one hundred registered participants and sixty talks over the two days. Participants came from all over the

UK and from further afield, including Algeria, China, the Czech Republic and Germany. In order to accommodate the large number of contributions, speakers were restricted to twelve minute talks, with three minutes for questions. The speakers, most of whom were PhD students or post-docs, did a universally excellent job of putting their work into context and presenting new results within these tight time constraints.

The presentations covered a wide range of topics, including gravitational waves, numerical relativity, the gravitational self force, neutron stars, cosmology, mathematical relativity, quantum gravity and higher dimensional black holes. To their great credit, all of the speakers made their talks understandable to conference participants outside of their immediate area of expertise.

As usual prizes were awarded for the best student talks, but this year in addition to the usual prize sponsored by Classical and Quantum Gravity, the IOP Gravitational Physics Group provided extra sponsorship which allowed prizes to be given for the best three student talks. The competition was very stiff, but in the end the first, second and third prizes went to Patricia Schmidt (Cardiff), Alexander Graham (Cambridge) and Gregory Ashton (Southampton) respectively, for their talks on “Modelling precession effects in binary black holes”, “Non-existence of black holes with non-canonical scalar fields” and “Gravitational wave searches from noisy neutron stars”. [ed: articles by these authors follow in the *Contributed Articles* section]

Classical and Quantum Gravity also sponsored a reception on the Monday evening, which was followed by a lively conference dinner in St Catharine’s College. We are very grateful to CQG for providing the wine for both of these events. We are also very grateful to the staff of St Catharine’s College and several PhD students from the Institute of Astronomy, without whom the meeting would not have run as smoothly as it did.

The next BritGrav will be hosted by the Gravitational Physics Group of the University of Birmingham on 20–21 April 2015. We look forward to seeing you all again there!

Prizes

2014 GPG Thesis Prize

The IOP's Gravitational Physics Group's thesis prize for 2014, co-sponsored by *Classical and Quantum Gravity*, has been awarded to Dr Anna Heffernan for her eloquently written thesis on the self-force problem in gravitational physics, and for her detailed calculation of the singular component of the divergent fields that arise.

Dr Heffernan obtained her PhD from University College Dublin, under the supervision of Prof. Adrian Ottewill. She currently works at ESA. Her thesis, entitled "The self-force problem: local behaviour of the Detweiler-Whiting singular field", is available on the arXiv ([arXiv:1403.6177](https://arxiv.org/abs/1403.6177)). You can also read an interview with Dr Heffernan on [CQG+](#).

Contributed Articles

The effect of timing noise on continuous gravitational wave searches

Gregory Ashton, University of Southampton

G.Ashton@soton.ac.uk

An international effort is currently underway to make the first detection of gravitational waves using laser interferometers. This includes the LIGO and Virgo detectors [1], which hope to be online by 2015 with advanced features, and the GEO detector [2] currently in operation. The advanced detectors are a huge step ahead of the previous generation and should be capable of making the first direct detection of gravitational waves.

In this article I want to share some recent work [3] to improve our understanding on how a phenomenon seen in pulsars called timing noise could impact on continuous gravitational wave (CW) searches. This is an important issue because if there are significant levels of timing noise in the CW signal then it may prevent us from being able to make a detection. In addition, if we observe a CW signal from a known pulsar, then comparing the timing noise in the CW and pulsar signals could give us new insights into neutron star physics.

General Relativity predicts that masses with time-varying quadrupole moments produce gravitational waves. For example, the first sources which we are likely to detect are binary systems where the loss of energy due to gravitational radiation will cause an inspiral and eventual merger of the objects. Gravitational waves have already been indirectly detected: the first evidence came from the 'Hulse-Taylor' binary pulsar system [4] and the discovery was subsequently awarded the 1993 Nobel prize.

Rapidly rotating isolated neutron stars (NSs) are a potential source of gravitational waves. These primarily lose rotational energy due to the torque caused by the electromagnetic radiation allowing some neutron stars to be observed as pulsars. Nevertheless, if they are non-axisymmetrically deformed, then they will also produce a continuous gravitational wave (CW) typically at twice their rotation frequency $f_{cw} = 2f_{rot}$ [5].

Gravitational waves are purely transverse with two polarisation states which we denote by + and x. In the reference frame of the source, the CW signal polarisations are

$$h_+ = A_+ \cos(\Phi(t)), \quad \text{and} \quad h_\times = A_\times \sin(\Phi(t)) .$$

For the interferometer detector, this looks like a time varying strain

$$h(t) = F_+ A_+ \cos(\Phi(t)) + F_\times A_\times \sin(\Phi(t)), \quad (1)$$

where the $F_{+,\times}$ and $A_{+,\times}$ are the antenna-pattern functions and amplitude parameters for the + and \times polarisation, respectively (for example, see Ref. [6] or for a review see Ref. [7]).

Data collected from the detector will contain noise in addition, hopefully, to a potential signal. We can decompose the detector output strain as the sum of the signal $h(t)$ and the noise $n(t)$

$$x(t) = n(t) + h(t) .$$

In general, the noise will be significantly stronger than the signal. To measure weak signals in the presence of noise, we can use matched filtering methods which compare the output of the detector $x(t)$ with a template for the signal $h(t)$. We can quantify how strong a signal $h(t)$ is compared to the noise floor of $n(t)$ by the signal-to-noise ratio (SNR) which we label as ρ . For a template that perfectly matches the signal, the SNR grows as (e.g. see Ref. [6])

$$\rho_S^2 \propto h_0^2 T_{obs}, \quad (2)$$

where T_{obs} is the observation time. This means that provided the template is a good match for the signal, matched-filtering methods are able to extract the weak CW signals from noisy data by searching over a sufficiently long time T_{obs} . Or to put it another way: integrating over a sufficiently long duration will eventually produce a sufficiently large SNR to be detected. The ability of the template to match the signal underpins the effectiveness of this process.

In order to use matched filtering methods, we need to have a model for the phase $\Phi(t)$ in equation Eq. (1). This process is aided by considering the population of NSs that we observe as *pulsars* first discovered by [8]. The mechanism, described independently by [9] and [10], that produces these pulses is that the neutron star has a strong dipolar magnetic field frozen into the crust. Along this dipole a beam of electromagnetic (EM) radiation is emitted. If this dipole is misaligned with the rotation axis then as the star rotates, it sweeps out beams of radiation. If these pass over the earth, then we observe them as short pulses of light at the rotation frequency of the neutron star.

The process of modelling signals from isolated neutron stars involves accounting for the intrinsic spin-down due to the EM and gravitational torques. Pulsar astronomers measure the time of arrivals (TOAs) of pulses correcting for various effects which occur during their journey between the source and detector; for more information on how this is done see [11]. The TOAs (in the solar-system barycentre) are fitted by a Taylor expansion in the phase:

$$\Phi(t) = \phi_0 + 2\pi \left(f_0(t - t_0) + \frac{\dot{f}_0}{2!}(t - t_0)^2 + \dots \right). \quad (3)$$

Here, the reference time t_0 and the coefficients for the phase, frequency and higher order terms $[\phi_0, f_0, \dot{f}_0, \dots]$ which best fit the observed TOAs is known as the *timing model*. A good timing model is able to exactly track the phase of a pulsar to within a single rotation over a duration of years.

The phase of CW signals will presumably be closely related to the phase of the EM signal from neutron stars observed as pulsars. This motivates the use of a Taylor expansion in the phase to generate templates for use in matched filtering. Since we said before that the ability of these templates to match the signal is critical to the success of CW detection, we should be careful to quantify when and how they can fail.

To evaluate the timing model pulsar astronomers define a *timing residual* as the difference between the observed TOAs and the timing model. In a well-timed model, for which there are no unmodelled effects, the timing residual will have a 'white' spectrum. However, two types of timing irregularities are observed in pulsars where we cannot get a well-timed model. The first, known as a glitch is a large and sudden increase in the rotation frequency, these happen predominantly in young pulsars and are useful to study the structure of neutron stars. The second is timing noise. This is a low-level structure that remains in the timing residual that we are as-of-yet unable to model with known neutron star physics. In Fig. 1 is an example of the timing residual for the Crab pulsar, a young pulsar which displays significant levels of timing noise.

Many attempts to interpret timing noise exist, almost all of these follow from the work of [13] who modelled it as a random walk in either the phase, frequency, or spin-down. The physical motivations for such random walks were investigated by [14], but no single process exists which explains all the observed effects. A recent and the most complete study on timing noise performed by [15] suggested that over sufficiently long time scales the structure in timing residuals

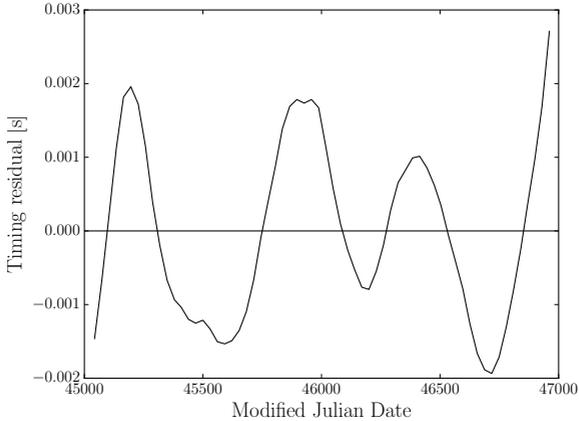


Figure 1: The timing residual for the Crab pulsar, collected using data from the Crab ephemeris [12]

have quasi-periodic features. This would suggest random walk models are not sufficient to explain timing noise and has prompted new discussions [16-18].

Regardless of the mechanism that produces timing noise or glitches, we must be careful to understand their implications when searching for CWs from neutron stars.

These timing variations are observed in pulsars, but here we are concerned with the corresponding variations that may exist in the CW signal. The mechanism that produces the electromagnetic radiation is thought to be coupled in phase to the crust of the neutron star from which we expect CWs to be emitted. Therefore it seems probable that variations in the EM signal may correlate to variations in any CW signal. If this is the case, then the Taylor expansion templates used in CW searches will not exactly match the signal due to the aforementioned timing irregularities. For glitches, this can result in a significant loss of SNR; as a result when searching for CWs from known pulsars we avoid periods of known glitches [19]. If the CW includes timing noise, the loss of SNR will be smaller, but could exist in all CW signals. This issue was first addressed by [20] where the author made crude estimates based on the random walk interpretation of timing noise. We now discuss the results of Ref. [3] where we have used observational data to quantify what effect, if any, timing noise could have on CW searches.

We begin by selecting the most in-depth observational data-set on a pulsar which displays significant levels of timing noise. The Jodrell bank observatory has recorded data on the Crab pulsar (PSR B0531+21) since 1982; they make this publicly available in a monthly ephemeris [12] hosted at www.jb.man.ac.uk/pulsar/crab.html. Each month they fit the observed TOAs by a Taylor expansion such as Eq. (3) up to the first-order in spin-down \dot{f}_0 . For the i th month, the resulting coefficients $\{f_i, \dot{f}_i\}$ are recorded along with a reference time t_i coinciding with the TOA of a pulse at the SSB. This choice of reference time ensures the phase $\phi_i = 0$ in Eq. (3).

The period of a month is considered to be short enough such that during each month the EM signal from the Crab is well-described by the first-order spin-down timing model. Between months however, there will be discontinuities that look like discreet jumps in the phase, frequency or spin-down. In Fig. 2, we illustrate such a jump in the frequency. The population of these ‘jumps’ is one way to describe the effect of timing noise for the Crab pulsar.

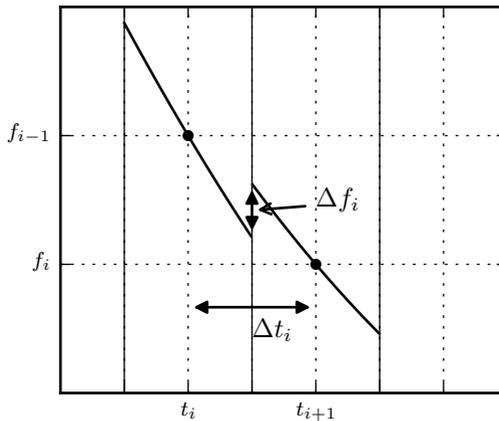


Figure 2: Illustration of the discontinuity in frequency between two lines of the Crab ephemeris. During each month, the frequency is a smooth Taylor expansion, which is illustrated by the two solid black lines centred on the reference time, t_i , and t_{i+1} . Between the two months, there is a jump in frequency resulting from timing noise.

Having a description of the timing noise in the EM signal we generated fake CW signals from the Crab ephemeris by assuming that the CW phase is

exactly twice the rotation phase. We construct the fake signals by injecting rectangular transient windows from each month long segment. The fake signal then looks like the illustration in Fig. 2, but with multiple months instead of just two. Ensuring that all windows are adjacent we have produced a CW signal that includes timing noise.

Now that we have generated fake CW signals we search for them using the current search methods. For this study we used the \mathcal{F} -statistic matched-filtering method [6] with Taylor expansion templates. This calculates the SNR for the fake CW signal containing timing noise.

A signal without timing noise will be 'perfectly phase matched' by a template since the timing noise describes the deviations from a single Taylor expansion. In Eq. (2), we labelled the squared SNR of such a signal as ρ_S^2 . If the squared SNR as measured from our signal *with* timing noise is ρ^2 , then we can quantify the effect of timing noise by the mismatch:

$$\mu = \frac{\rho_S^2 - \rho^2}{\rho_S^2}. \quad (4)$$

So, in summary our method consists of: selecting a period of data from the Crab ephemeris; generating fake CW signals which include the timing noise described by the ephemeris; and finally we retrieve these signals and quantifying the effect of timing noise by the mismatch μ .

The Crab pulsar has been the subject of several targeted searches by LIGO and Virgo that have used Taylor expansion templates: for example the so-called 'narrow-band' searches in [19] and [21]. The first thing we did was to check the mismatch predicted by our method for the Crab signal in these searches. We found that in both cases the mismatch was less than 1%. This suggests that for these searches, the effect of timing noise was negligible.

Having confirmed that timing noise was not an issue for these specific searches, we then asked the question: when could it become an issue? From Eq. (2), we know that for perfectly matched signals, looking at longer observation periods should increase detectability. However, intuitively we can imagine that the 'amount' of mismatch might increase with the observation period. We investigated this by measuring the resulting mismatch for different epochs and total observation times. Fig. 3 shows the averaged (over epochs) mismatch $\langle\mu\rangle$ as a function of total observation time T_{obs} .

From this evidence we find that that the mismatch increases non-linearly with the observation time. From a power law fit (the dashed line in Fig. 3) we find that the average mismatch scales with the observation time as:

$$\langle \mu \rangle \sim T_{obs}^{2.88} .$$

If this non-linear increase exists in real CW signals, then the linear increase of square signal to noise ratio of Eq. (2) will compete with the degradation due to timing noise. From the power-law fit in Fig. 3, we estimate that for observation periods greater than 600 days the SNR will begin to decrease (with greater observation duration) due to the effects of timing noise.

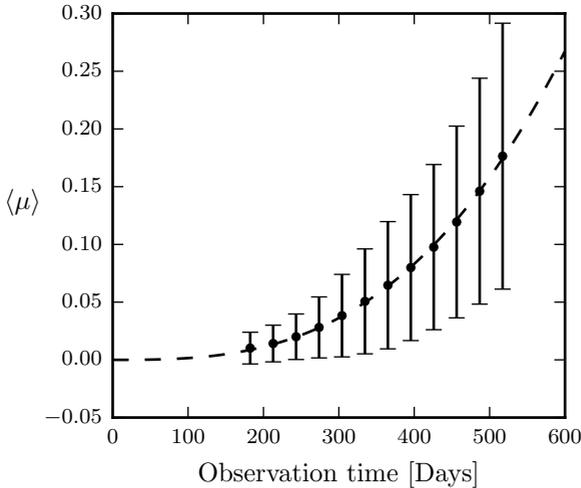


Figure 3: The averaged mismatch (see Eq. (4)) as calculated from the Crab ephemeris against the observation period. This shows that the mismatch increase for longer searches. The dashed line shows the result of the power law fit.

We have considered only one realisation of timing noise and have made the assumption that timing noise in the EM signal is also observed in the CW signal. However, these results should also give a good estimate for cases where

the level of timing noise is similar in the CW signal, but not exactly the same. From this we suggest that current searches are not in danger of the adverse effects of timing noise. But, we should be careful when increasing the observation duration with hopes of increasing the detectability of signals. In the future we intend to extend this study by using the observed pulsar population to understand the implications for other types of CW searches.

References

- [1] G. M. Harry and LIGO Scientific Collaboration, *Class. Quantum Grav.*, **27** 084006 (2010)
- [2] H. Grote and LIGO Scientific Collaboration, *Class. Quantum Grav.*, **27** 084003 (2010)
- [3] G. Ashton, D. I. Jones, R. Prix, *arXiv:14108044*, (2014)
- [4] R. A. Hulse and J. H. Taylor, *ApJ*, **195** L51-53 (1975)
- [5] S. L. Shapiro and S. A. Teukolsky, *John Wiley and Sons, Inc.* (1983)
- [6] P. Jaranowski, A. Królak and B. F. Schutz, *Phys. Rev. D* **58** 063001 (1998)
- [7] R. Prix in vol. 357 of *Astrophysics and Space Science Library*, pp 651 (2009)
- [8] A. Hewish et al, *Nature*, 217:709-713 (1967)
- [9] F. Pacini, *Nature*, 216:567-568, (1967)
- [10] T. Gold, *Nature*, 218:721-732 (1968)
- [11] A. Lyne and F. Graham-Smith *Cambridge University Press* (2012)
- [12] A. G. Lyne, R. S. Pritchard and F. Graham-Smith, *MNRAS*, 265:1003 (1993)
- [13] P. E. Boynton and E. J. Groth, *ApJ* (1972)
- [14] J. M. Cordes and G. S. Downs, *ApJS*, 59:343-382 (1985)
- [15] G. Hobbs, A. G. Lyne and M. Kramer, *MNRAS* 402(2):1027-1048 (2010)
- [16] A. Lyne et al, *Science* 329:408 (2010)
- [17] J. M. Cordes, *ApJ* 775:47 (2013)
- [18] D. I. Jones, *MNRAS* 420(3):2325-2338 (2012)
- [19] B. Abbott et al, *ApJ*, 683:L45-49 (2008)
- [20] D. Jones *Phys. Rev. D* **70** 042002 (2004)
- [21] LIGO Scientific Collaboration et al, *Phys. Rev. D*, **91** 022004

Black holes and scalar hair

Alexander A. H. Graham, DAMTP, University of Cambridge

A.A.H.Graham@damtp.cam.ac.uk

It is one of the most remarkable features of general relativity that when an in principle arbitrarily complicated system undergoes gravitational collapse to form a black hole the end state is described entirely by at most three parameters. This result, which has become known as the black hole uniqueness theorem, is without doubt one of the crowning achievements of the golden age of relativity. In essence, it tells us that we have a complete mathematical understanding of the structure of a black hole once it has settled down in its environment, which is virtually without parallel in science. To paraphrase Chandrasekhar, black holes are not only amongst the most exotic objects in the universe but are also amongst the simplest.

The original formulation of the black hole uniqueness theorems, or non-hair theorems, are concerned with the existence of stationary black hole solutions to the Einstein-Maxwell equations. Essentially, they state the following: the only stationary, asymptotically flat, single black hole solution of the Einstein-Maxwell equations is the Kerr-Newman black hole [1,2]. This was proven via a series of results in the late 1960s and 1970s. The logical structure of the proof, which is not quite the same as its historic structure, starts with Hawking's proof in 1972 that a stationary, asymptotically flat black hole has topologically spherical horizons (the topology theorem), and is either static or axisymmetric (rigidity theorem) [3]. The static case was essentially resolved a few years earlier by Israel [4], who showed that the only such vacuum black hole solution is the Schwarzschild solution (Israel's original proof made certain unnecessary technical assumptions, but these were later removed [5]). This was quickly generalised to show similarly that the Reissner-Nordström solution is the only static, electrovac black hole [6].

The stationary and axisymmetric case is, of course, much more difficult, but first Carter [7] and later Robinson [8] were able to demonstrate that the only such black hole is the Kerr solution. The argument was originally given by Carter using a perturbative argument; Robinson provided the full non-linear generalisation of this argument via his discovery of a highly non-trivial identity between solutions of the field equations. The proof was generalised a few years later to the electrovac case [9] -- the key step was the appropriate generalisation of Robinson's identity. Although there are still one or two unresolved technical

issues -- the rigidity theorem make the unnecessarily restrictive assumption that the spacetime is analytic¹, and the case of multi-black holes is still not fully resolved² -- it is basically established lore. How to extend these results to higher dimensions or to spacetimes which are asymptotically anti-de Sitter is still an open problem, as in both cases no simple generalisation of the topology theorem exists.

Although the no-hair theorems are largely resolved in the electrovac case it is still an active area of research to extend these results to different matter models, such as for scalar fields or non-Abelian gauge fields. While, of course, there is no experimental evidence for any long-ranged forces beyond the electromagnetic and gravitational forces it is a common suggestion in cosmology that additional matter fields, often scalar fields, exist as an explanation for dark energy. If they do exist then whether or not they affect the structure of black holes, and thereby whether or not their existence could in principle be deduced from the observational study of black holes, essentially boils down to answering if a version of the no-hair theorems hold. It is therefore not just a question of academic interest, but potentially could be a key astrophysical constraint on models of dark energy in a regime of gravitational physics very different from cosmological observations. Besides, it is of obvious academic interest to wonder how far the no-hair theorems can be extended.

It should be noted that when one discusses if a matter model coupled to gravity possesses a no-hair theorem there are two senses in which the term is used. One the one hand, the term can be used in a strong sense to mean that if one searches for stationary, asymptotically flat black hole solutions to the Einstein equations coupled to a given matter model then no black hole solutions exist beyond the Kerr solution. That is, the black hole cannot support any non-trivial excitation of the field at all. The no-hair theorems for scalar fields are normally of this form.

¹ There has been some recent progress in removing this assumption, but not enough to consider it solved. The best results known presently are due to Alexakis et al [10], who were able to remove this assumption for spacetimes perturbatively close to the Kerr spacetime.

² In the static case it is known that no static, vacuum multi-black holes exist [11], and it is also known that the only static, electrovac multi-black holes are contained within the Majumdar-Papapetrou spacetimes. In the stationary case the case of two black holes has been ruled out [12], but the case of more than two black holes is still open.

This is also a weaker notion of uniqueness, which allows for the existence of non-Kerr black holes in the theory, but demands that if all the globally conserved charges of the theory are specified the spacetime is uniquely fixed. An example of theories where uniqueness holds in this sense is when we take a (massless) scalar field dilatonicly coupled to electromagnetism. Various results are known which show that once the black holes mass and charge is specified the spacetime is uniquely fixed, even though it is not in general given by the Reissner-Nordström solution [2]. It is known that uniqueness can fail in both senses for appropriately chosen matter. The best example are non-Abelian gauge fields [13], and a great deal of research now exists for this and related matter models.

In this article we will focus on the black hole uniqueness theorems which hold for a single, real scalar field, φ . We will only allow the scalar field action to depend on the first derivative of the scalar field, but will in principle allow it to be non-canonical. That is, we take the action of the scalar field to be given by

$$S = \int d^4x \sqrt{-g} P(\varphi, X), \quad (1)$$

where $X = -\frac{1}{2} \nabla_\alpha \varphi \nabla^\alpha \varphi$. A canonical scalar field with self-interaction potential $V(\varphi)$ is given by the choice $P = X - V(\varphi)$. The equations of motion are easily found to be

$$\nabla_\alpha (P_{,X} \nabla^\alpha \varphi) + P g_{ab},$$

where a comma denotes a partial derivative. The energy-momentum tensor is

$$T_{ab} = P_{,X} \partial_a \varphi \partial_b \varphi + P g_{ab}.$$

This action includes both quintessence and K-essence scalar field theories, and so is appropriate for examining if scalar field models of dark energy admits hairy black holes. It does not cover Horndeski's theory though, which is the most general scalar-tensor theory possessing second order equations of motion [14].

For the general action of this form until recently the only no-hair results known are due to Bekenstein [15], who studied static, spherically symmetric, asymptotically flat black hole solutions to this system. He showed that the only such black hole was the Schwarzschild solution provided the action obeyed the conditions

$$P(\varphi, X) \leq 0, \text{ and } P(\varphi, X)_{,X} > 0 .$$

Notice that for a canonical scalar field these conditions reduce to the requirement that the potential is bounded from below $V(\varphi) \geq 0$. This result in the canonical case was also established independently by Sudarsky [26] and Heusler [17].

If one restricts to a canonical scalar field a second set of results are known, which were proven by Bekenstein in 1972 [18]. He showed that the only stationary, asymptotically flat black hole solution was the Kerr solution provided that the potential obeyed

$$\varphi V_{,\varphi} \geq 0 , \text{ or } V_{,\varphi\varphi} > 0 .$$

This also assumed that the potential was such that $\varphi \rightarrow 0$ at spatial infinity, so it does not apply to Higgs-like potentials. It does though rule out new rotating black holes unlike the later results, which only applied to static, spherically symmetric black holes³. The proof, which only involves use of the scalar field equation of motion, is also very general. For instance, it is immediately obvious that this result also rules out black hole solutions of the Einstein equations with scalar hair when an electromagnetic field is included.

Recently, a new no-hair theorem has been given which extends these results to general scalar field actions of the form of Eq. (1) [19]. The proof, which is a straightforward generalisation of Bekenstein's original argument, follows by multiplying the equation of motion by φ and integrating over a region Σ from outside the black hole horizon to infinity. Upon integrating the first term by parts we arrive at the identity

$$- \int_{\Sigma} (P_{,X} \nabla_{\alpha} \varphi \nabla^{\alpha} \varphi - \varphi P_{,\varphi}) \sqrt{-g} d^4x + \int_{\partial\Sigma} \varphi P_{,X} \nabla^{\alpha} \varphi dS_{\alpha} = 0$$

where $dS_{\alpha} = n_{\alpha} d\sigma$ and n^{α} is the normal of the boundary $\partial\Sigma$. The boundary term is composed of two parts: one evaluated at infinity, and the other evaluated at the horizon. The contribution from infinity vanishes provided that $\varphi \rightarrow 0$ as $r \rightarrow \infty$, as

³ It is still an open problem to show that no stationary, asymptotically flat black holes with canonical scalar hair exist if $V(\varphi) \geq 0$. Indeed, it is still an open problem to show no hairy, static black holes exist with this restriction on the potential, even without the assumption of spherical symmetry.

the solution is asymptotically flat which requires the field to decay appropriately at infinity. The horizon contribution can also be shown to vanish provided the field and its first derivatives are regular (which is required for the energy-momentum tensor to be well-defined) by the Cauchy-Schwarz identity [18]. Hence, any hairy black hole solution must satisfy the following identity:

$$\int_{\Sigma} (P_{,x} \nabla_{\alpha} \varphi \nabla^{\alpha} \varphi - \varphi P_{,\varphi}) \sqrt{-g} d^4x = 0 . \quad (2)$$

If we, for now, assume that φ has no dependence on time then $\nabla_{\alpha} \varphi$ is spacelike outside the horizon. This is because $\nabla_{\alpha} \varphi \nabla^{\alpha} \varphi = g^{ij} \partial_i \varphi \partial_j \varphi$, and as by the Cotton-Darboux theorem [1] we may choose coordinates to diagonalise the spatial part of the metric this is a sum of squares. Hence, if either of the following conditions is satisfied

$$P_{,x} > 0, \quad \varphi P_{,\varphi} \leq 0, \quad \text{or} \quad P_{,x} < 0, \quad \varphi P_{,\varphi} \geq 0$$

then the integrand is everywhere positive or negative respectively, and so Eq. (2) can only be satisfied provided that $\varphi = \text{constant}$, and the black hole supports no scalar hair.

Note that the second condition covers fields which violates the null energy condition, which are not covered by Bekenstein's proof. This immediately rules out black holes with a massless ghost field, and a ghost field with a tachyonic potential. The theorem can also be used to show that the well known ghost condensate model and the DBI models admit no stationary, asymptotically flat black hole solutions. Note also that the argument generalises when the cosmological constant is non-zero.

While these results apply to a very large class of scalar field theories they do not apply to Horndeski's theory, since the action is not included within Eq. (1). Recently, there has been some progress with understanding the structure of static, spherically symmetric black hole solutions for the shift-symmetric subset of the theory. Hui and Nicolis [20] gave an argument that generically they cannot admit scalar hair. This was later examined by Sotiriou and Zhou [21], who showed that for a special choice of the Horndeski action this argument could be evaded and hairy black holes can exist. They were recently found numerically in Ref. [22].

While the previous argument is straightforward and makes few assumptions on the black hole one might notice that there is a subtle assumption which may be unjustified. This assumption is in fact common to all previous no-hair theorems for scalar fields in the literature. It is that we assumed that the

spacetime geometry being stationary implies the scalar field has no dependence on time. For a scalar field theory in which the Lagrangian explicitly depends upon φ this is indeed the case. However, if the Lagrangian only depends upon the gradient of φ , so that $P = P(X)$, (which includes a massless, canonical scalar field) then only the gradient of φ enters the energy-momentum tensor, and the Einstein equations imply only that $\nabla_\alpha \varphi$ is independent of time, not φ itself. A scalar field profile of the form $\varphi = t + f(r)$ would be compatible with this, and so we might worry that there is a missing family of stationary, hairy black hole solutions with time-dependent scalar hair not covered by the previous results. This is reinforced by the recent discovery of stationary, hairy black hole solutions in Horndeski's theory with a time-dependent scalar field precisely of this form, even though generically no time-independent scalar hair is admitted for these theories [23,24]

In the case of scalar field theories of the form of Eq. 1, this cannot occur, as we can show in this case no such black holes exist [25]. The proof applies equally well to rotating black holes, but for simplicity we will give the argument in the static case. In this case we may choose coordinates (t, x_i) so that the metric is diagonal, and a simple calculation shows that $R_{tx_i} = 0$. The key step of the argument is to examine the off-diagonal Einstein equations⁴. If the scalar field is independent of time, these give no information, but if $\partial_t \varphi \neq 0$, they imply that $\partial_{x_i} \varphi = 0$, so that $\varphi = \varphi(t)$. That is, the only time-dependent scalar field profile consistent with the geometry is when φ depends solely upon time. Moreover, from the requirement that the energy-momentum tensor is independent of time we must in general have that φ can only depend linearly upon time, so the only acceptable scalar field profile in general is

$$\varphi = \alpha t + \beta ,$$

where α and β are constants. It is easy to check that this is not in general compatible with asymptotic flatness unless $\alpha = 0$, and so the black hole admits no time-dependent scalar hair.

In this article we have only touched upon a few aspects of the subject, and there are many additional areas one can explore. One of the more interesting areas, in light of the current interest in modifications to general relativity, is what light they shed on the black hole structure of different theories of gravity? For

⁴ This is the step which does not work in Horndeski's theory.

instance, it was realised long ago by Hawking [26] that the original no-hair theorems for a scalar field can immediately be used to show that the only stationary black hole solution of the vacuum equations of Brans-Dicke theory is the Kerr solution, a result which has recently been extended to general scalar-tensor theories of gravity [27]. Can these results be used for other theories of gravity? Work is underway to establish if this is the case.

Acknowledgements

The work described here was done in collaboration with Rahul Jha. I thank John Barrow, Anne-Christine Davis, Sam Dolan and Elizabeth Winstanley for helpful discussions, and Harvey Reall for drawing attention to an error in an earlier version. I am supported by the STFC.

References

- [1] S. Chandrasekhar, *Oxford University Press* (1983)
- [2] P. T. Chruściel, J. L. Costa and M. Heusler, *Living Rev. Relativity* **15**, 7 (2012)
- [3] S. W. Hawking, *Commun. Math. Phys.* **25**, 152-166 (1972)
- [4] W. Israel, *Phys. Rev.* **164**, 1776-1779 (1967)
- [5] D. C. Robinson, *Gen. Rel. Grav.* **8**, 695-698 (1977)
- [6] W. Israel, *Commun. Math. Phys.* **8**, 245-260 (1968)
- [7] B. Carter, *Phys. Rev. Lett.* **26**, 331-333 (1971)
- [8] D. C. Robinson, *Phys. Rev. Lett.* **24**, 905-906 (1975)
- [9] P. O. Mazur, *J. Phys. A* **15**, 3173-3180 (1982)
- [10] S. Alexakis, A. D. Ionescu and S. Klainerman, *Commun. Math. Phys.* **299**, 89-127 (2010)
- [11] G. L. Bunting and A. K. M. Masood-ul-Alam, *Gen. Rel. Grav.* **19**, 147-154 (1987)
- [12] G. Neugebauer and J. Hennig, *J. Geo. Phys.* **62**, 613-630 (2012)
- [13] P. Bizon, *Phys. Rev. Lett.* **64**, 2844-2847 (1990)
- [14] G. W. Horndeski, *Int. J. Theor. Phys.* **10**, 363-384 (1974)
- [15] J. D. Beckenstein, *Phys. Rev. D* **51**, R6608-R6611 (1995)
- [16] D. Sudarsky, *Class. Quantum Grav.* **12**, 579-584 (1995)
- [17] M. Heusler, *Cambridge University Press* (1996)
- [18] J. D. Beckenstein, *Phys. Rev. Lett.* **28**, 452-455 (1972); *Phys. Rev. D* **5**, 1239-1246 (1972); *Phys. Rev. D* **5**, 2403-2412 (1972)
- [19] A. A. H. Graham and R. Jha, *Phys. Rev. D* **89**, 084056 (2014)
- [20] I. Hui and A. Nicolis, *Phys. Rev. Lett.* **110**, 241104 (2013)
- [21] T. P. Sotiriou and S-Y. Zhou, *Phys. Rev. Lett.* **110**, 251102 (2014)
- [22] T. P. Sotiriou and S-Y. Zhou *Phys. Rev. D* **90**, 124063 (2014)

- [23] E. Babichev and C. Charmousis, *JHEP* **1408**, 106 (2014)
- [24] T. Kobayashi and N. Tanahashi, *Prog. Theor. Exp. Phys.* **2014**, 073E02 (2014)
- [25] A. A. H. Graham and R. Jha, *Phys. Rev. D* **90**, 041501(R) (2014)
- [26] S. W. Hawking, *Commun. Math. Phys.* **25**, 167-171 (1972)
- [27] T. P. Sotiriou and V. Faraoni, *Phys. Rev. Lett.* **108**, 081103 (2012)

Untangling precession: Modelling the gravitational wave signal from precessing black hole binaries

Patricia Schmidt, California Institute of Technology and Cardiff University
pschmidt@ligo.caltech.edu

The coalescence of two stellar mass black holes (BH) is regarded as one of the most promising sources for the first gravitational-wave (GW) detection with ground-based interferometric detectors such as advanced LIGO [1,2]. Our current understanding of the physics of such compact binaries is limited by electromagnetic (EM) observations. Gravitational-wave observations, however, will allow us to 1) directly identify their intrinsic properties (mass and spins) and 2) complement EM information for binaries submerged in a matter-rich environment to obtain a more complete understanding of the physics, and especially the strong-field regime of General Relativity (GR).

The current detection strategy, *matched filtering* [3], and the subsequent determination of the source parameters rely on theoretical knowledge of the emitted gravitational waveforms. It is therefore crucial to obtain an accurate and complete description of the GW signal that covers the entire binary evolution.

GW emission causes the orbital separation to gradually decay until the two black holes merge into one. Roughly, this evolution undergoes three distinct stages, which are commonly modelled in different ways. Firstly, the inspiral phase when the two black holes are far apart. In this regime, black holes are commonly approximated as point particles without internal structure and their motion and the GW signal are well described within post-Newtonian (PN) theory. During the later stages of the inspiral $v \simeq c$ and the merger, the PN approximation becomes invalid and the full nonlinear field equations have to be solved numerically. Finally, the two objects merge into a single perturbed black hole before settling down to the stationary Kerr solution during the ringdown phase; the GWs can either be obtained directly from numerical relativity binary simulations or via semi-analytical methods in perturbation theory. Commonly, in order to semi-analytically describe the complete GW signal from the inspiral, through merger to the ringdown (IMR), we utilise some combination of numerical and analytical information.

However, combing the relevant information and developing simple, computationally fast waveform models is a non-trivial task, especially for generic binaries. Consider two spinning BHs with arbitrarily oriented spin angular momenta \vec{S}_i or their dimensionless counterparts (for $i=1,2$) $\vec{\chi}_i = \vec{S}_i/m_i^2$ and arbitrary mass ratio $q = \frac{m_2}{m_1} \geq 1$ in a quasi-circular orbit; the total mass $M = m_1 +$

m_2 scales out of the modelling problem. Such binaries span a seven-dimensional intrinsic parameter space, which significantly complicates the modelling task. Two important, well-understood subclasses of BH binaries are non-spinning binaries, which are entirely described by q , and aligned-spin binaries (two Kerr black holes with \vec{S}_i parallel to the orbital angular momentum \hat{L}), which are described by q and the two spin magnitudes S_i . Waveforms across the parameter space for these two subclasses have been successfully modelled in the past [4-6]. For non-spinning and aligned-spin binaries, the inspiral dynamics are entirely confined to a spatially fixed orbital plane. When we allow for arbitrary orientation of the spins, however, the inspiral dynamics are no longer confined to the orbital plane as relativistic couplings between the orbital angular momentum and spin angular momenta induce precession of the orbital plane as well as precession of the spins. The precession motion on top of the inspiral motion is directly reflected in the GW signal in the form phase and amplitude modulations [7,8]. The magnitude and time evolution of these modulations are set by the physical parameters of the binary system, and therefore all seven parameters need to be taken into account. Moreover, precession also makes the problem of combining numerical with PN information more difficult and therefore the construction of complete waveform model becomes vastly more complex. During the past four years, we have been focusing on developing a general framework, which allows us to build waveforms from precessing black hole binaries in a systematic and geometrically meaningful way, which I will describe in detail in the subsequent sections.

Untangling precession

Quadrupole-alignment

Modelling the complete GW signal from the inspiral through merger to the ring down of precessing compact binaries in quasi-spherical orbits has been a long standing challenge due to the structural complexity of the waveforms and the high-dimensional parameter-dependence. However, it turns out that the complex structure can be significantly simplified by analysing the gravitational waveforms in a non-inertial coordinate frame. In particular, we can find a frame that co-rotates with the bulk precessional motion following the evolution of the orbital plane of the binary system determined by the time evolution of \hat{L} . We determine this particular frame, the *quadrupole-aligned* (QA) frame, by maximising the ($\ell = 2, |m| = 2$)-modes of the GW signal and comparison with PN results shows that we are indeed tracking the orbital plane [9].

In the QA-frame, the strong modulations due to the coupling between L and the spins are partially suppressed, yielding smoother GW modes with only

very little remaining structure. In fact, those “co-precessing” waveforms resemble waveforms emitted by aligned-spin or non-spinning binaries to a very high degree as illustrated in Fig. 1.

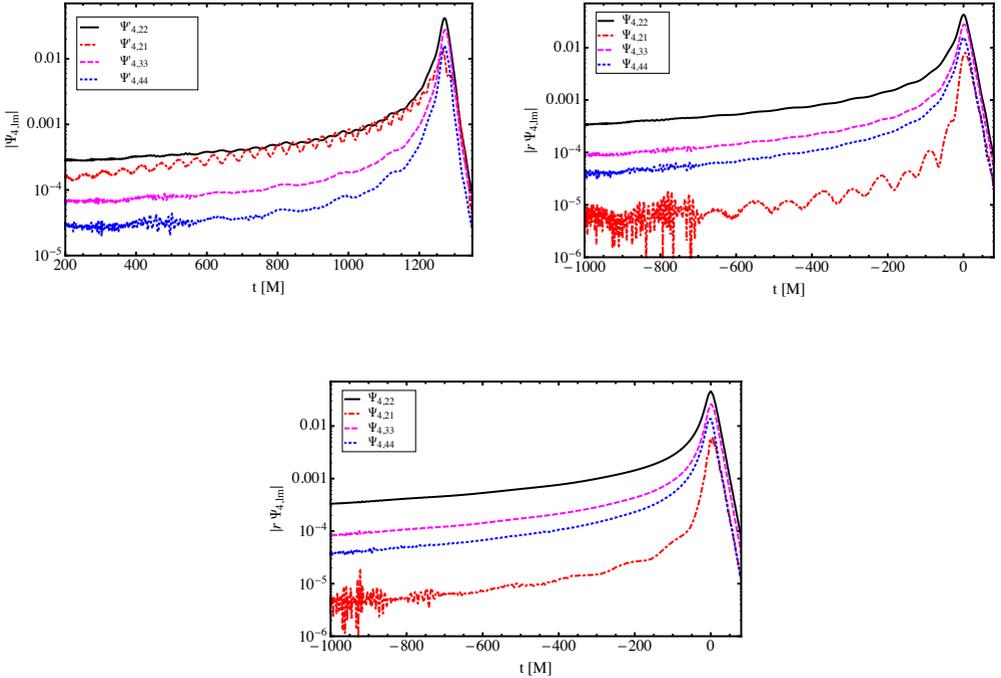


Figure 1: The three panels show GW modes from (top left) a precessing numerical relativity simulation of mass ratio $q=3$. Strong modulations are clearly visible; (top right) the same modes in the QA-frame and (bottom) the same modes for a non-spinning binary of the same mass-ratio. The similarity between the QA and non-spinning modes is striking.

By transforming into this non-inertial frame, we track the precession of the orbital plane to a good approximation and are therefore able to almost directly observe the secular motion of the binary during the late inspiral up to the merger. Moreover, the amplitudes of the GWs appear rather smooth for arbitrary binary orientations, and we also restore the hierarchical mode structure observed

in non-precessing binaries. As the QA-waveforms look almost like the ones of spin-aligned or non-spinning binaries, we are led to an important insight: the inspiral and the precession dynamics are approximately decoupled. Under this frame transformation, we have removed the evolution of the orbital plane, and therefore split the problem of modelling these feature rich precessing waveforms into two smaller tasks, namely into 1) describing the precession of the orbital plane and 2) describing the secular evolution through the merger.

A mapping between precessing and non-precessing binaries

Firstly, we aim to find a description of the secular motion we have uncovered in the QA-frame. In order to test the correspondence suggested above, we need to map the unwrapped precessing waveforms onto corresponding non-precessing ones. Since computationally expensive numerical relativity simulations only allow us to sample a few discrete points in the binary parameter space, PN provides an ideal test bed to quantify the mapping hypothesis for a large number of binary configurations. In our analysis, we find that the unwrapped waveforms indeed agree best with those non-precessing (NP) waveforms that have very similar spin components parallel to the orbital angular momentum, i.e., $\vec{S}_i^{nonPrec} \simeq (\hat{L} \cdot \vec{S}_i) \hat{L} \equiv \vec{S}_{i||}$, and the same mass ratio as the precessing configuration. This can be understood as follows: The phase evolution of aligned-spin binaries is well known to high order in PN theory. A priori, our best guess is to compare the precessing waveforms in the QA-frame to aligned-spin waveforms, which have the same amount of spin in their parallel components. Alternatively, we may also combine these two parallel spin components into one effective total spin [10] and distribute it equally between the two black holes. We find that the best agreement can indeed be achieved for spin values very close to the theoretically expected ones [11]. Moreover, this agreement is not only true for post-Newtonian inspiral waveforms but extends through merger, which is crucial for the modeling of complete waveforms within this framework. During merger, however, it becomes more approximate and has largely broken down during the ringdown [11,12].

With those ingredients, we can now build a robust and simple framework to model the complex GW signal from precessing binaries by performing the inverse procedure: waveforms from precessing black hole binaries are well approximated by “twisting up” aligned-spin waveforms with an appropriate description of the precession dynamics [11], which schematically read as

$$h^{Prec}(t; q, \vec{\chi}_1, \vec{\chi}_2) \approx \mathbf{R} h^{nonPrec}(t; q, \vec{\chi}_1^{Prec}, \vec{\chi}_2^{Prec}), \quad (1)$$

where the operator \mathbf{R} contains the precession dynamics. This is illustrated in Fig.2 for an inspiral waveform. This procedure works all the way up to merger and is therefore a simple and ideal framework to construct complete inspiral-merger-ringdown waveform models [11]. The missing ingredient, however, is an accurate description of the precession dynamics across the parameter space.

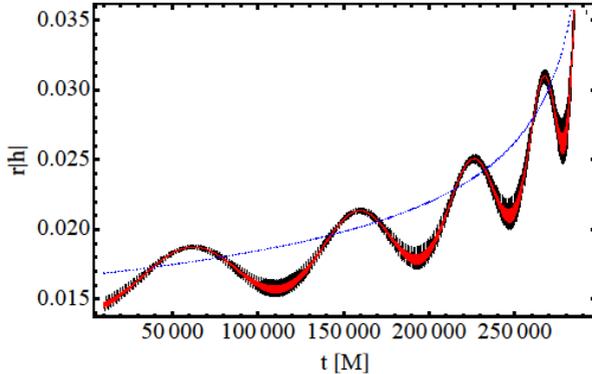


Figure 2: The panel shows the “mock” precessing waveform (red) obtained by twisting up an aligned-spin waveform (blue) with the appropriate rotation operator constructed via quadrupole alignment. For comparison, the black curve is the “true” precessing waveform computed by solving the corresponding PN equations.

An effective description of the precession dynamics

At leading-order, the evolution of the orientation of the orbital plane is encoded in two polar angles $(\iota(t), \alpha(t))$ which define the orientation of \hat{L} on the unit sphere, and constitute the main ingredients of the operator \mathbf{R} in Eq. (1). Obtaining accurate expressions across the parameter space proves difficult due to their dependence on *all* physical parameters. Even for pure PN inspiral signals analytic solutions of the precession equations can only be found for special cases [7]. However, since the secular and precession dynamics approximately decouple, it might also be possible to separate the parameter dependence accordingly. In fact, we have already shown that the secular evolution is accurately described by one respectively two spin parameters for a fixed mass ratio -- the inspiral depends only weakly on the spin components orthogonal to \hat{L} . Is it possible to also reduce the number of dependent parameters to find approximate descriptions of the precession angles? [13]

Similarly to the effective inspiral spin, we find a combination of spin parameters that allows us to define an *effective precession spin* χ_p . The leading-order PN precession equations [7] show that the evolution of \vec{L} is driven by particular spin projections into the instantaneous orbital plane, i.e.,

$$\dot{\vec{L}} \propto \left[\left(2 + \frac{3q}{2} \right) \vec{S}_1 + \left(2 + \frac{3}{2q} \right) \vec{S}_2 \right] \times \hat{L} =: A\vec{S}_{1\perp} + B\vec{S}_{2\perp},$$

where $\vec{S}_{i\perp} = \vec{S}_i \times \vec{L}$. Further, we observe that these in-plane spin projections continuously change their relative orientation within the orbital plane, and that their magnitudes show only small variations on the long inspiral timescale and can therefore be approximated as constant. Taking this into account, we postulate that the average precession exhibited by a generic system can be captured by only one effective precession spin [13],

$$S_p := \frac{1}{2} (AS_{1\perp} + BS_{2\perp} + |AS_{1\perp} - BS_{2\perp}|) \equiv \max(AS_{1\perp}, BS_{2\perp}),$$

where we choose $S_{i\perp}$ to be the magnitude of the in-plane projection of the i -th spin at the initial moment in the evolution. Since we commonly define binary configurations using the dimensionless spin parameters χ_i , we define the effective precession spin

$$\chi_p := \frac{S_p}{Bm_2^2}.$$

The choice of assigning the precession spin to the larger black hole is motivated by the fact that precession effects are more strongly influenced by $BS_{2\perp}$.

We have tested the hypothesis by comparing the waveforms of generic binaries with those from binaries with the reduced parameter dependence. Explicitly, the mapping is given by

$$h(t; q, \vec{\chi}_1, \vec{\chi}_2) \mapsto h'(t; q, \chi_{1\parallel}, \chi_{2\parallel}, \chi_p).$$

A good agreement in this comparison provides evidence that the precession dynamics during the inspiral is well captured by only one additional spin parameter. We have investigated this mapping for 10,000 randomly chosen generic binary configurations for the mass ratios $q=1,3,10$ for a discrete set of binary orientations and signal polarisations yielding 300,000 comparisons per mass ratio. The results are illustrated in Fig.3, which shows the fantastic

agreement in the form of a cumulative distribution function (CDF). Our results suggest that the precession angles are well described by their average obtained by collapsing the mass-weighted in-plane spin projections into χ_p and assigning it to the heavier black hole. We note, however, that this is only true if the inspiral rate is captured correctly, which is another crucial ingredient in this simple and geometric framework to model the waveforms of precessing binaries.

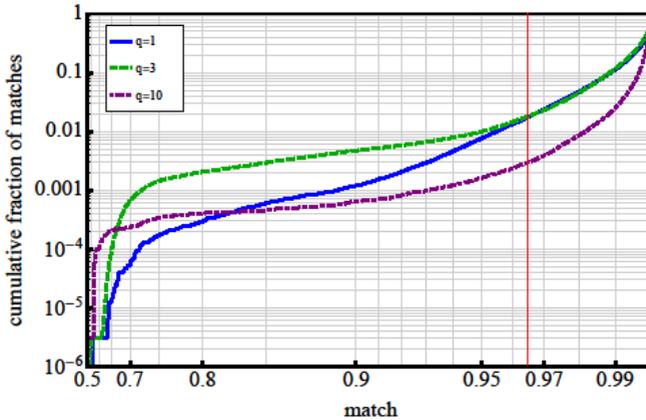


Figure 3: The cumulative distribution function of matches computed for 10,000 random binary configurations for the comparable mass ratios $q=1$ (solid blue), $q=3$ (dashed green) and $q=10$ (dot-dashed purple). The red vertical line indicates the arbitrary threshold of 96.5%. We find that for $q=1,3$, less than 2% of all computed matches are below this threshold; for $q=10$, only 0.3%.

Discussion and concluding remarks

Over the course of the past four years, we have developed a simple framework to model the complete waveforms of generic compact binaries in quasi-spherical orbits. Our starting point was the introduction of a particular co-precessing frame, which allowed us to observe that the precession dynamics and the underlying secular evolution decouple approximately. We have then shown that the secular evolution can be uniquely identified with the secular motion of a spinning, non-precessing counterpart, and we have provided the appropriate mapping, which is applicable through merger, making it particularly useful for developing complete waveform models. The complex problem of modelling precessing waveform then factorised into two smaller tasks, namely into

modelling the waveforms of spinning, non-precessing binaries, and such models already exist [4-6], and into finding a description of the precession itself to "twist up" the non-precessing waveforms to create precessing ones. The construction of a model for the precession dynamics, however, is a non-trivial problem. Despite its complexity, we have shown that the precession dynamics observed in a generic configuration is well mimicked by its average, captured by only one additional spin parameter, the effective precession spin, which makes the task of developing complete inspiral-merger-ringdown models of precessing waveform much more tractable. This general framework has been successfully used to construct a simplified inspiral waveform model [14], a precessing effective-one-body waveform model [15] and a closed-form phenomenological IMR waveform model [16].

Acknowledgements

This work has been carried out in collaboration with Mark Hannam, Frank Ohme and Sascha Husa. I would like to thank Geraint Pratten and Mark Hannam for useful comments.

References

- [1] S. Waldman *The Advanced LIGO Gravitational Wave Detector* (2011)
- [2] G. M. Harry, *Class. Quantum Grav.*, **27** 084006 (2010)
- [3] K. S. Thorne, pp 330-458 *Cambridge University Press* (1987)
- [4] P. Ajith et al, *Phys. Rev. Lett.* **106** 241101 (2011)
- [5] L. Santamaria et al *Phys. Rev. D* **82** 064016 (2010)
- [6] A. Taracchini et al *Phys. Rev. D* **82** 064016 (2010)
- [7] T. A. Apostolatos et al *Phys. Rev. D* **86** 6274-6297 (1994)
- [8] L. E. Kidder, *Phys. Rev. D* **52** 821-847 (1995)
- [9] P. Schmidt et al, *Phys. Rev. D* **84** 024046 (2011)
- [10] P. Ajith, *Phys. Rev. D* **84** 024046 (2011)
- [11] P. Schmidt, M. Hannam and S. Husa, *Phys. Rev. D* **86** 104063 (2012)
- [12] L. Pekowsky et al, *Phys. Rev. D* **88** 024040 (2013)
- [13] P. Schmidt, F. Ohme and M. Hannam, *preprint* (2014)
- [14] A. Lundgren and R. O'Shaughnessy, *Phys. Rev. D* **89** 04021 (2014)
- [15] Y. Pan et al, *Phys. Rev. D* **89** 084006 (2014)
- [16] M. Hannam et al, *Phys. Rev. Lett* **113** 151101 (2014)

Group Committee

Nils Andersson (Southampton)	na@maths.soton.ac.uk
Adam Christopherson (Florida) <i>Newsletter Editor</i>	achristopherson@gmail.com
Timothy Clifton (QMUL) <i>Honorary Secretary</i>	t.clifton@qmul.ac.uk
Davide Gerosa (Cambridge) <i>Student Representative</i>	d.gerosa@damtp.cam.ac.uk
Giles Hammond (Glasgow)	giles.hammond@glasgow.ac.uk
Ik Siong Heng (Glasgow) <i>Treasurer</i>	Ik.Heng@glasgow.ac.uk
Bangalore Sathyaprakash (Cardiff) <i>Chair</i>	b.sathyaprakash@astro.cf.ac.uk
Juan A. Valiente-Kroon (QMUL)	j.a.valiente-kroon@qmul.ac.uk
Silke Weinfurter (Nottingham)	Silke.Weinfurter@nottingham.ac.uk

Information about the group and membership

To find out more about the group, you can visit our website:

<http://gp.iop.org>

If you would like to contribute an article to the next newsletter, please contact Adam Christopherson at

achristopherson@gmail.com

In order to join the group, for members, the easiest way is to go to

<http://members.iop.org/login.asp>

After logging in, click on “My Groups”, and from there it is straightforward to add/remove groups.

For IOP members without internet access, the simplest way to join the group is to amend your membership renewal form. Alternatively, write to the membership department (address below). Note that membership of all IOP groups is now free.

If you are not a member of the IOP and would like to join, you should visit

<http://www.iop.org/Membership/index.html>

where it is possible to apply online. Applications are also accepted by writing to the IOP.

This newsletter is also available on the web and in larger print sizes

The contents of this newsletter do not necessarily represent the views or policies of the Institute of Physics, except where explicitly stated.

The Institute of Physics, 76 Portland Place, W1B 1NT, UK.

Tel: 020 7470 4800

Fax: 020 7470 4848