

Entropy in the teaching of thermal physics

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Caution: Entropy



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Statistical Physics

An Entropic Approach



 WILEY

Coming out in May 2013

Summary

- Uncertainty is better than disorder
- Coarse graining of the environment underlies entropy production
- Irreversible processes are the norm
- Information entropy maximisation is dynamical

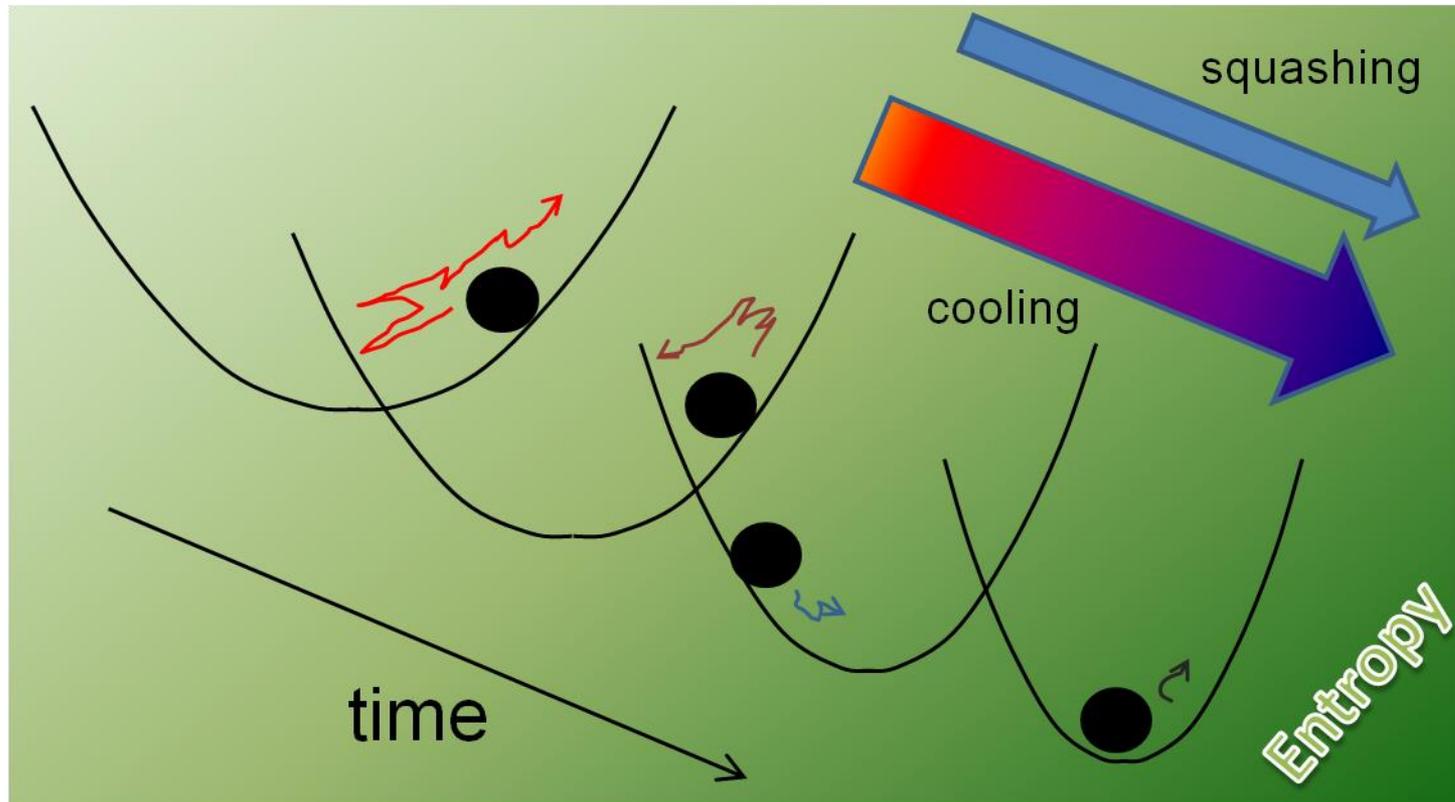
1. Blah

2. Blah

3. Blah

4. etc.

Thermodynamic processes and entropy production



'Disorder' is not the ideal paradigm



Is this what we mean by entropy change?

Is 'rearrangement' the arrow of time?

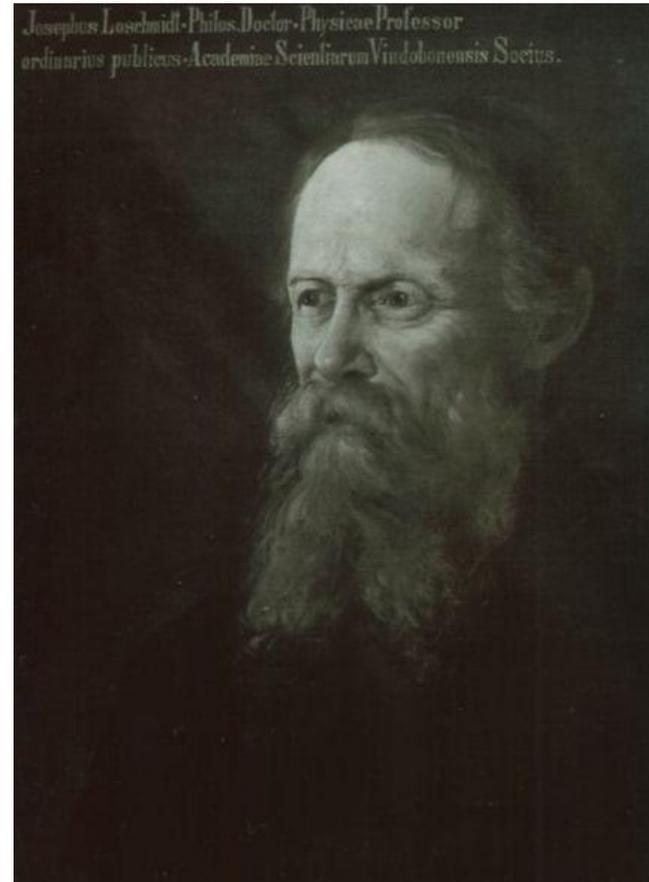
'Uncertainty' is better



The future is uncertain due to lack of data about the present. And so is the past.

Loschmidt's reversibility paradox

- Mechanics is time-reversal invariant
- How can we get an entropy that always increases with time?
- For an ill-specified system, uncertainty increases both ways.



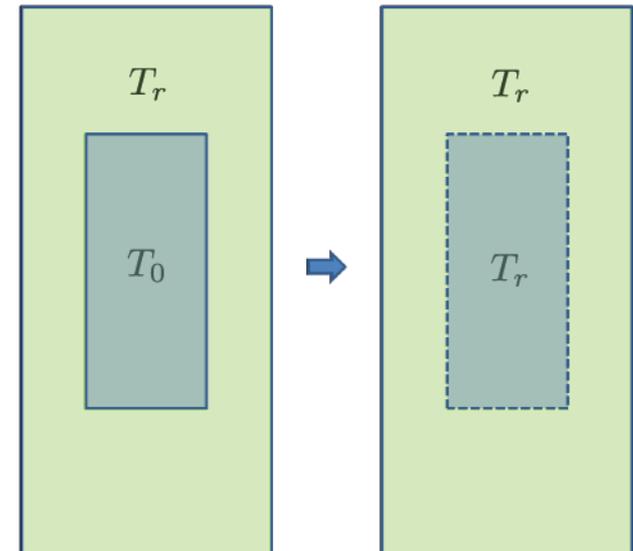
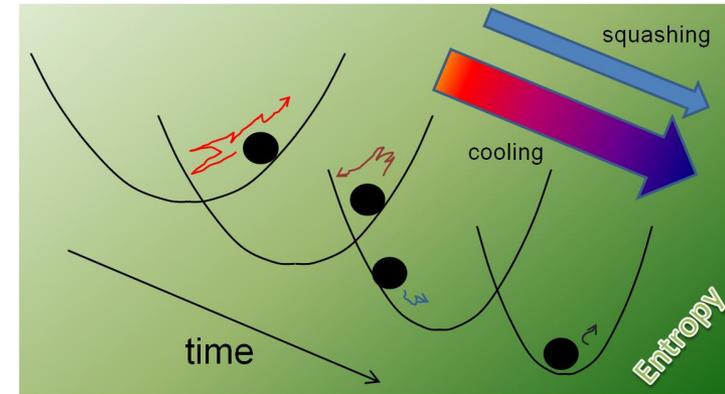
The untidy room analogy

- Start with a room+contents+child
 - Close the door
 - Ask generic questions
 - Guess what is happening
-
- Entropy (uncertainty) increases until a maximum is reached, conditioned by what we can find out remotely.

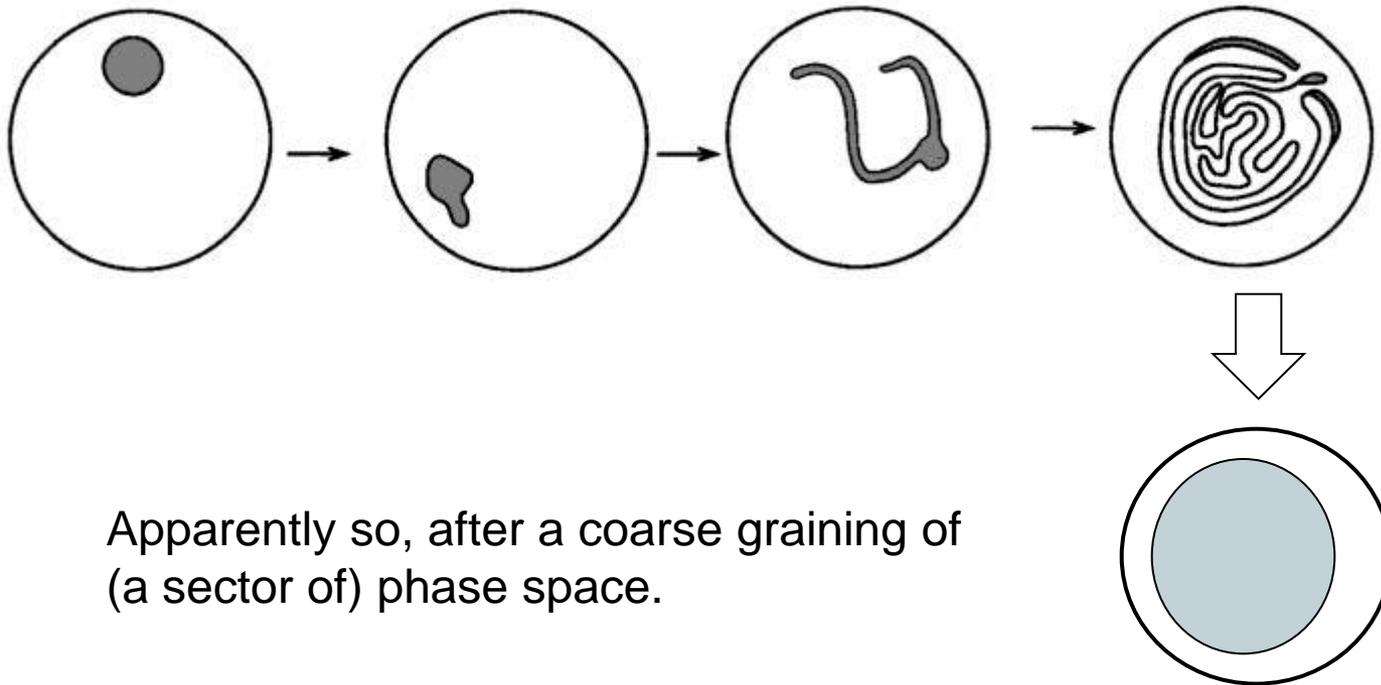


Uncertainty about the present is synonymous with coarse graining

- Prime example: the reservoir/heat-bath/environment
- A system may or may not be coarse grained



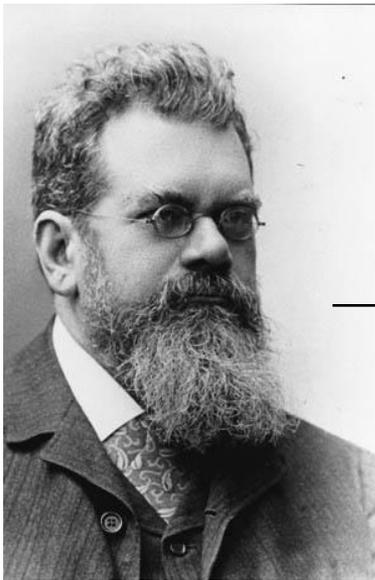
Does nonlinear dynamics produce entropy?



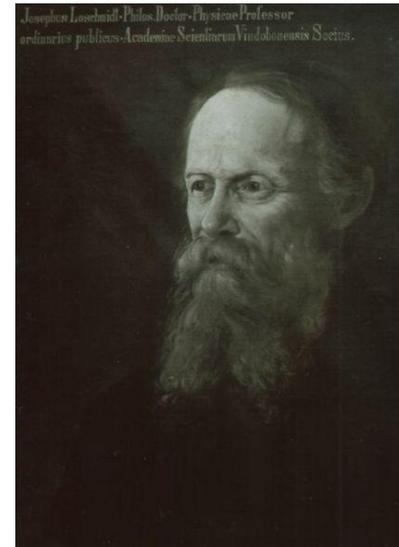
Apparently so, after a coarse graining of
(a sector of) phase space.

Entropy production

- Time-reversal invariance broken through coarse graining in the model dynamics
- Entropy increase is the evidence of this breakage at the macroscopic level



Loschmidt,
I've derived
the H-theorem!



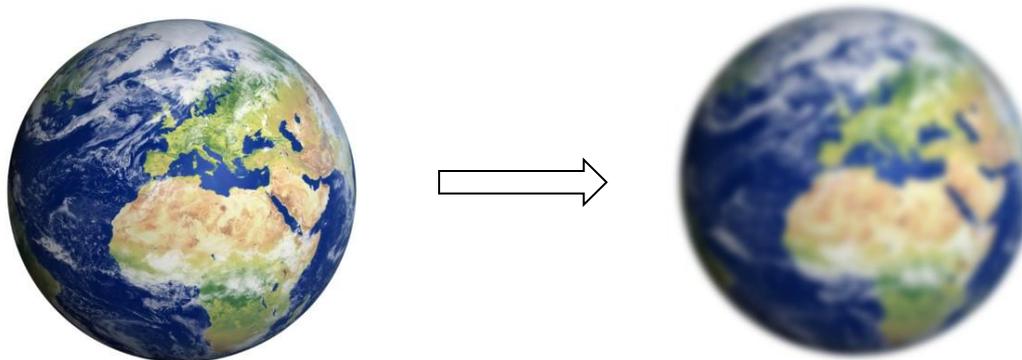
Not very
happy

Summary so far:

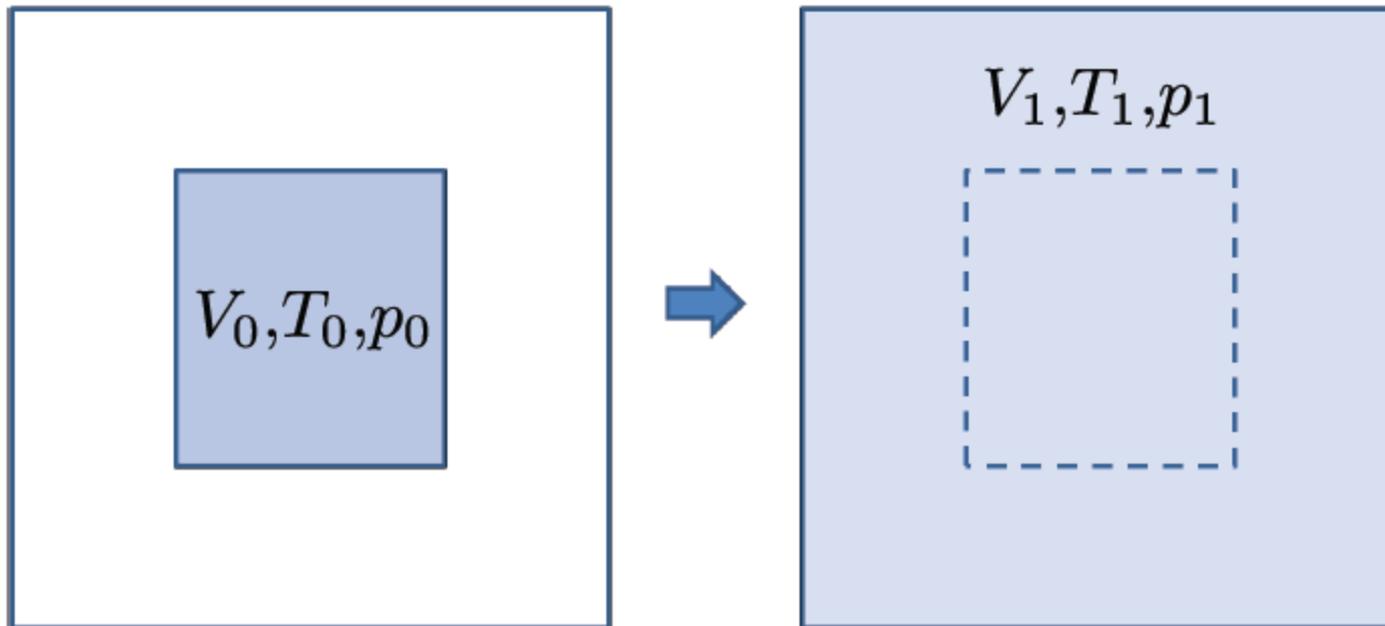


Caution: Entropy

- Making analogies with disorder can be confusing
- Entropy ties more easily to uncertainty
 - an increase is a loss of information
- Intuitively, a consequence of *dynamics* starting from an incomplete specification of the universe



Next: irreversible processes are the norm



$$S(T_0, N, V_0) = Nk \ln \left(\frac{(kT_0)^{3/2}}{\hat{c}N/V_0} \right) \quad \text{Ideal gas}$$

$$\Delta S = S(T_0, N, V_1) - S(T_0, N, V_0) = Nk \ln(V_1/V_0)$$

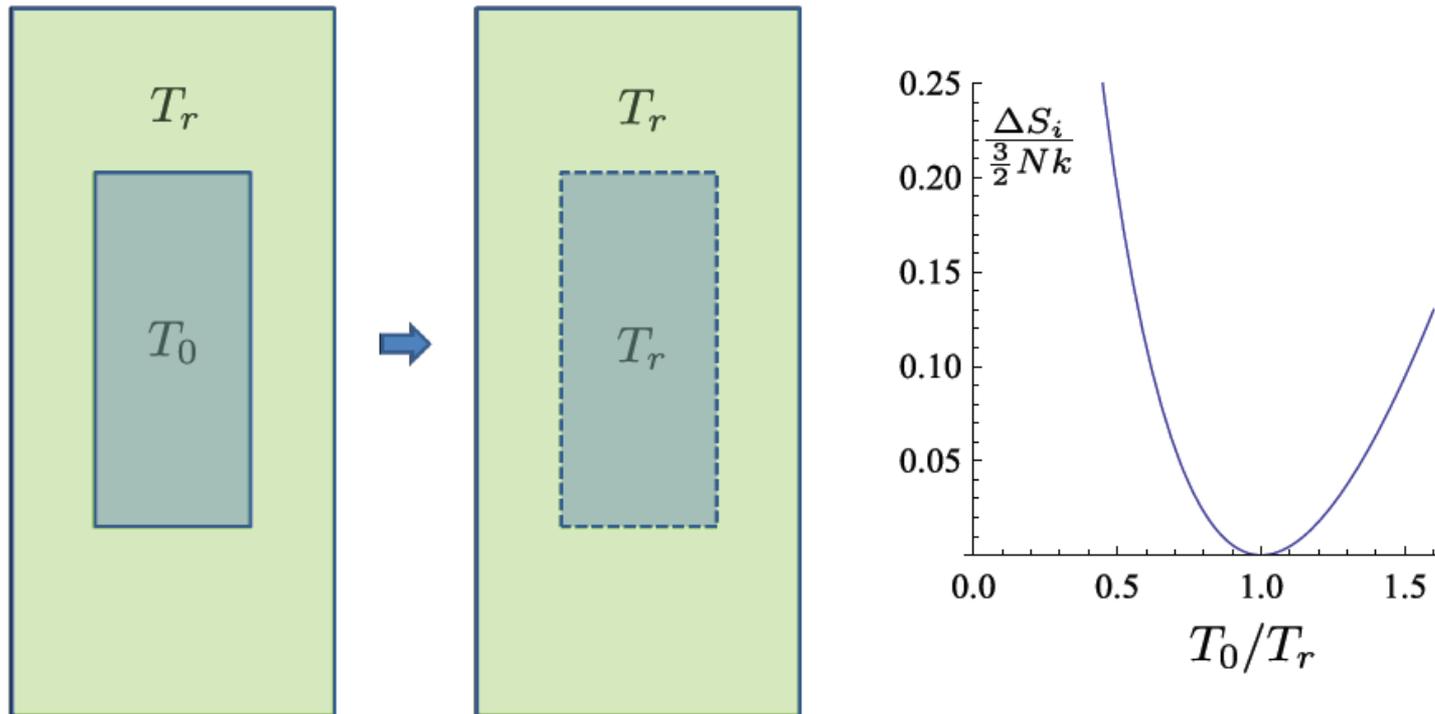
Entropy production is the norm

$$dS = \frac{dQ}{T_r} + dS_i$$

- This is a dynamical equation, not just a mathematical relationship

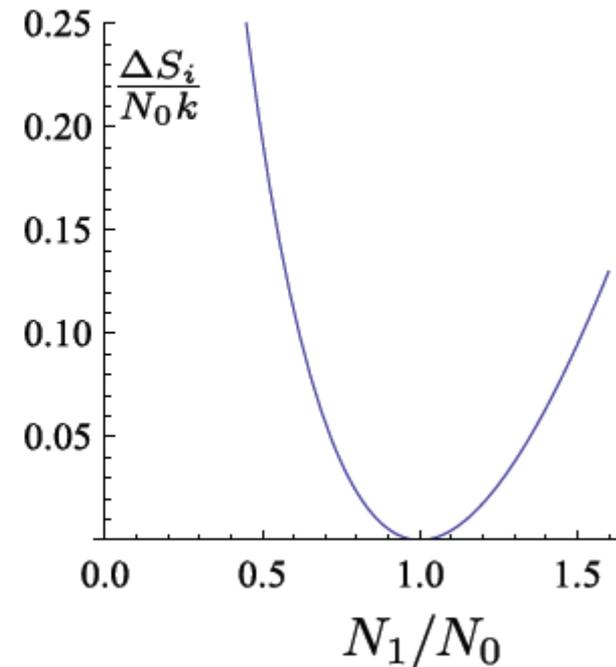
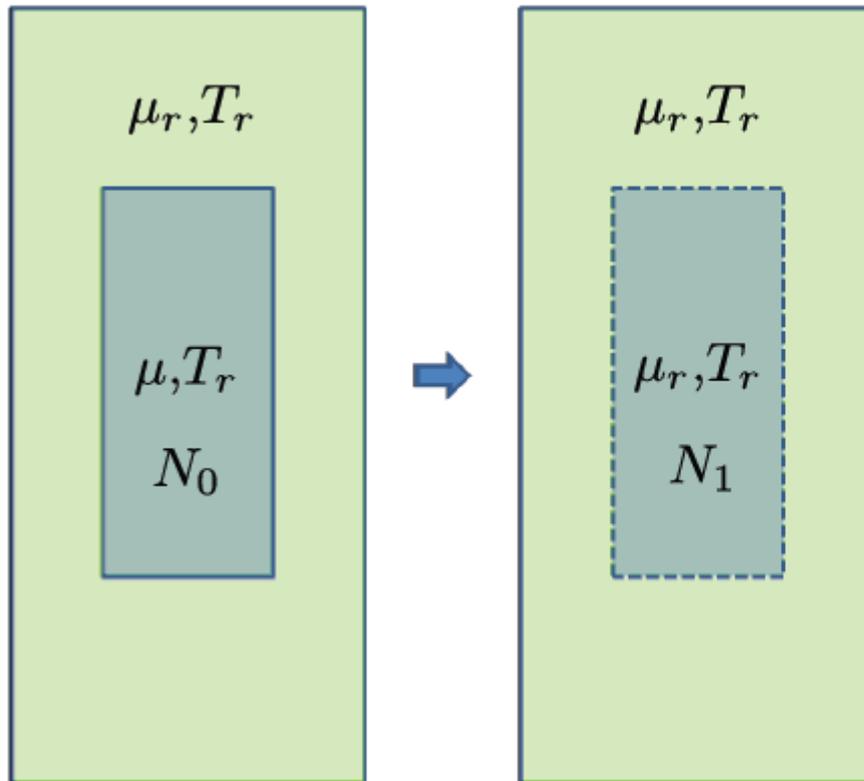
$$\Delta S_{\text{tot}} = \Delta S_i \geq 0 \quad \text{second law}$$

Spontaneous, nonquasistatic heat exchange



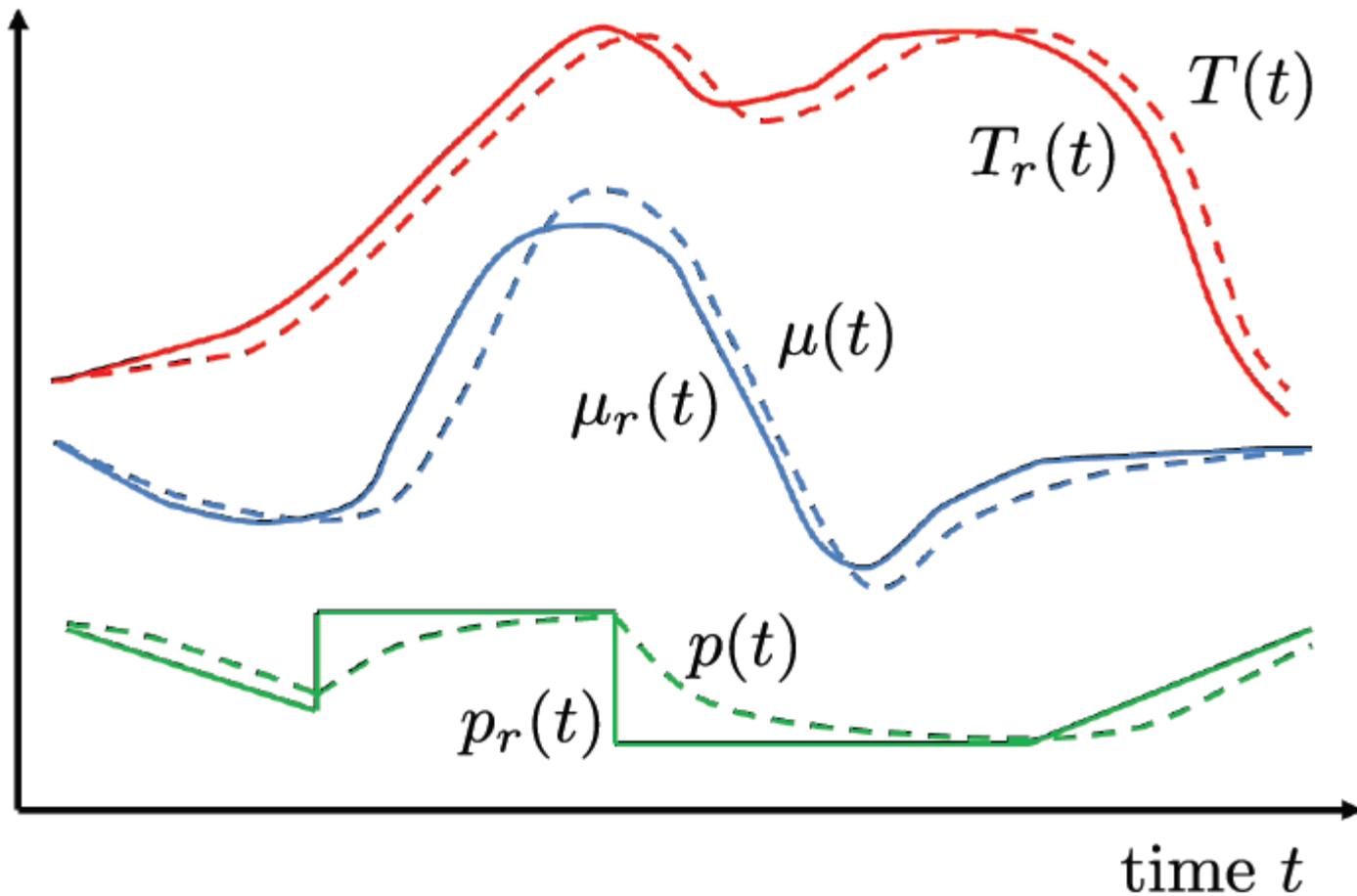
$$\Delta S_i = \frac{3}{2} Nk \left(\ln \frac{T_r}{T_0} - \frac{T_r - T_0}{T_r} \right)$$

Spontaneous, nonquasistatic particle exchange



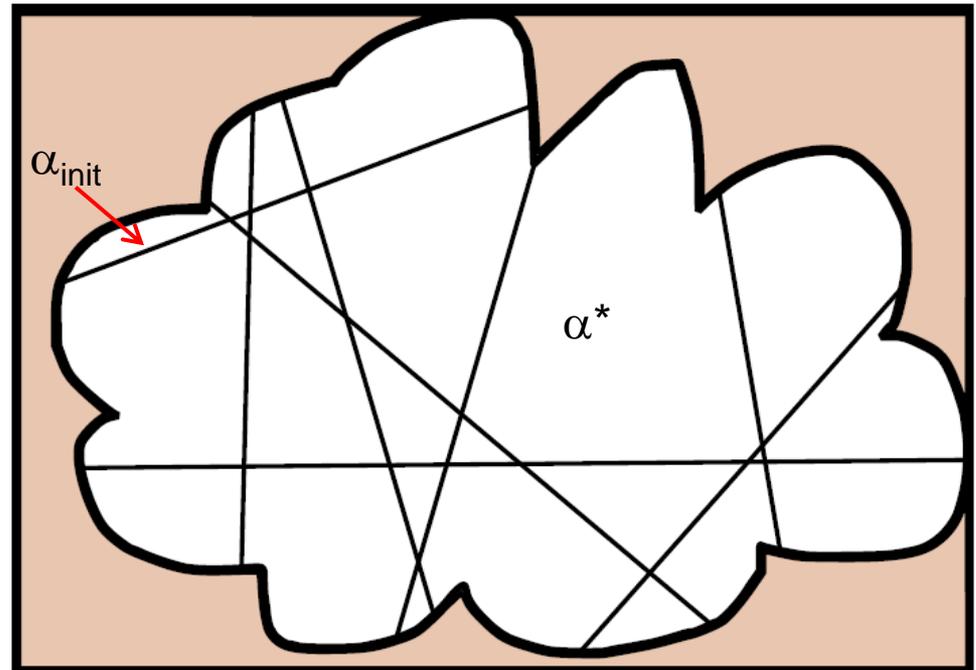
$$\Delta S_i = k(N_1 - N_0) + kN_0 \ln(N_0/N_1)$$

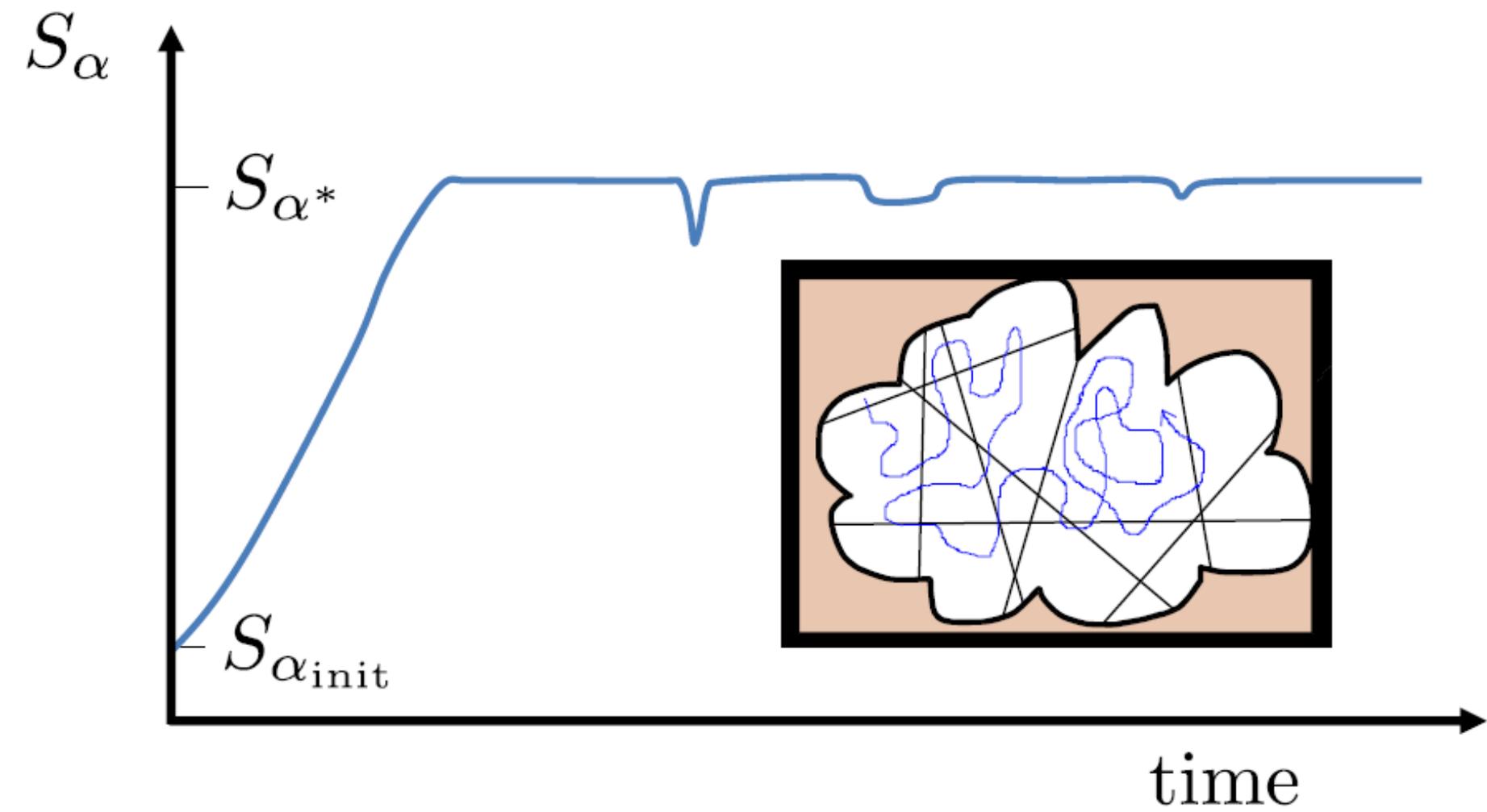
Irreversibility and time-lag



Boltzmann entropy maximisation is dynamical

- Picture of phase space, carved up into macrostates α
- Random dynamical exploration



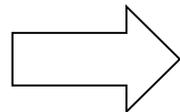


Lastly:

Information entropy maximisation is dynamical

- Hard to motivate the maximisation of Shannon entropy $S_I = -k \sum_i P_i \ln P_i$ subject to constraints.

$$\frac{\partial}{\partial P_j} \left(-k \sum_i P_i \ln P_i - \lambda \sum_i P_i - \lambda' \sum_i E_i P_i \right) = -k \ln P_j - k - \lambda - \lambda' E_j = 0$$



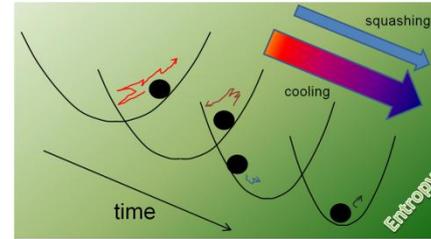
$$P_i = Z^{-1} \exp(-\beta E_i)$$

Dynamical evolution of total entropy

$$\begin{aligned}\frac{dS_{\text{tot}}}{dt} &= \frac{dS_I}{dt} + \frac{dS_r}{dt} = \frac{dS_I}{dt} + \frac{1}{T_r} \frac{d\langle E_r \rangle}{dt} \\ &= \frac{dS_I}{dt} - \frac{1}{T_r} \frac{d\langle E \rangle}{dt} = \frac{d}{dt} (S_I - T_r^{-1} \langle E \rangle) \geq 0\end{aligned}$$

- Logic of unbiased assessment, or just a consequence of the dynamics of probability?

In conclusion



- Analogies like order \Rightarrow disorder can be confusing
- Definiteness \Rightarrow uncertainty stands up better
 - ties in with coarse graining of the environment
- Dynamics should be emphasised
- Irreversibility and entropy production are normal
- Constrained information entropy maximisation is the dynamical saturation of total uncertainty:
 - the view from behind the closed door
- Thanks for listening!

