IOP Institute of Physics

Professor Sir Adrian Smith's review of post-16 mathematics provision in England
POST-16 maths requirements for physics

Summary

It is our view that A-level physics students would benefit from studying mathematics beyond GCSE in a way that

- phases well with their physics studies;
- develops a fluency in the mathematical techniques that will be needed in higher education courses;
- provides them with mathematical tools beyond GCSE level so that they can better understand the physics they are studying and also get a more authentic view of the ways that physicists analyse problems before choosing their next step.

The current structures make it difficult to insist that all students should study, for example AS level mathematics. And, even if they did, the current AS level in mathematics does not satisfy all the needs of the above. As a potential solution we make two suggestions, the latter of which we see as more feasible: a) a short course in mathematics that provides students with the tools to study physics in a mathematical way or b) a framework of baccalaureate-style qualifications at 18 which would allow for much more efficient phasing of content in mathematics and physics.

1. *What is your view of the mathematics and quantitative skills young people have when they enter employment or higher education?*

a. The IOP's *Mind the Gap*[1] report in 2011 found, amongst other things, that:

   i. Nearly a half of physics and engineering undergraduates found the mathematics of their course more difficult than they expected.

   ii. 55% of tutors felt that their students were not well enough prepared for the mathematics on their course.

   iii. A large proportion (92%) of academics felt that a lack of fluency in mathematics was an obstacle to students achieving their full potential in the long term, and more than four in five (85%) agreed that a lack of fluency affected their department's ability to deliver an optimal programme of study.

b. Furthermore, the 2012 SCORE report *Mathematics Within A-level Science 2010 Examinations*[2] found that many of the mathematical questions in A-level physics exams were

[1] http://www.iop.org/publications/iop/2011/file_51933.pdf
[2] http://www.score-education.org/media/10033/score%20maths%20in%20science%20summary%20report.pdf

repeated versions of low level algebraic manipulations. There was little variety from year to year and very few examples of questions that required the use of logarithms or graphs – although these were in the specifications. Therefore, students did not need to practice or develop fluency in these techniques.

c. The same report found that there was a paucity of multi-step calculations. Whilst a physics A-level paper in the 1990s and a paper in 2010 might both include algebraic manipulations to find, for example, the period of an orbit, the 2010 question gave more guidance to the examinees.

d. A-level physics students need opportunities and encouragement (through the assessment scheme) to develop and practice the use of mathematical techniques in physical situations so that they acquire a deep and lasting fluency in those techniques.

2. What basic level of mathematics and quantitative skills do businesses and higher education need school leavers to possess? Do leavers meet those levels now?

e. The requirement to progress to a degree in physics is both an A-level in physics and an A-level in mathematics. Physics at university is quite different from many other university courses in the respect that it requires two identified A-level subjects. A number of engineering courses have similar requirements.

f. Even for students who have achieved good grades in both physics and mathematics, the main comment from higher education physics departments is that students lack 'fluency' in mathematics and numerical techniques.

g. Furthermore, students in both physics and mathematics found it hard to set up or apply mathematical models of physical situations.

h. The concern relating to the last two responses in question 1 is not that papers have got easier over time, it is that students do not have to practice multi-step calculations or the use of more advanced techniques in order to take the exams. Therefore they leave school without the fluency in mathematics that they would have had if they had been required to practice those techniques.

3. Are there sufficient school leavers with advanced mathematics and quantitative skills for the needs of business and higher education?

i. Currently, more than 24,000 school leavers have both physics and mathematics at A-level (the requirement to start a physics degree or some engineering degrees). Physics departments are able to recruit effectively from this pool, but a larger pool would allow them to take on even greater numbers of students.

j. However, a bigger pool would increase the number of students aiming for engineering degrees with the dual qualification.

k. Furthermore, it would be beneficial to the system if more students were taking physics and mathematics A-levels and then moving on to technical routes or into employment.

l. There are also a small number of students who choose physics but not maths at A-level who come to make their university application and find they are unable to apply for physics (and some engineering) courses because they do not have mathematics. Better advice needs to be provided to students and schools to inform them of the implications of the choices they make when selecting both GCSEs and A-levels.

4. Does post-16 maths provision need to change to better meet current requirements? How?

m. To make the most of the existing requirements of A-level physics and, as we propose below, the inclusion of ideas from calculus, students would benefit hugely from formally studying mathematics beyond GCSE level. The new A-level will cover all of the requirements that we list in the Appendix and most of them are covered by the AS level (with the exceptions of 6 and 9).

n. Of the 28,592 18 year olds who sat physics A-level in 2015, 26,887 (94%) of them had an AS in mathematics and 24,274 (85%) of them also took maths A-level. Furthermore, 7,502 of them took A-level Further Mathematics.

o. It is not possible, in the current environment, to insist that all A-level physics students should take either AS mathematics or A-level mathematics. There is no doubt that this requirement would provide better support for the 6% of students who tend to take physics without any mathematics beyond GCSE. However, within a system of free choice at A-level, this requirement would put physics A-level in the unique position of requiring an additional, specified, full subject within the A-level options – thereby restricting the choice of those choosing physics.

p. It would be desirable, therefore, to have both a systemic structure and an appropriate qualification that will support physics A-level students to study enough mathematics to support their physics studies.

q. It would also be desirable that the maths content phases well with the physics content.

r. Furthermore, the phasing should be the same for all students. The situation would not be made better by the students in a physics class having four different experiences of mathematics by February of Year 12 (maths AS level, maths A-level, maths + further maths A-level and – as suggest propose below - a new short course). Efforts to improve the mathematical skills of physics students should not risk reducing the take-up of physics.

5. How do you see those requirements changing in the next 5-10 years?

s. In 2013, the IOP set up a Curriculum Committee to determine, from scratch, the requirements of an A-level in physics. It is likely that the committee will recommend some changes to the mathematical requirements in order to:

    i. provide students with an authentic experience of physics and a taste of what physics and engineering would be like at university, thereby helping them to make informed choices about courses or occupations after A-level. The *Mind the Gap* report found that there were mathematics undergraduates who had rejected physics as a degree because the impression of school physics was that it was about reading and learning rather than solving problems. This highlighted the inauthentic nature of their experience of physics at school.

ii. develop their fluency in mathematics – especially in the use of mathematics to solve physical, engineering and technical problems.

iii. demonstrate the power of mathematical solutions to physical problems that use mathematics beyond GCSE level.

t. In order to address the points in paragraph 1, a reimagined physics A-level will require material beyond the current mathematics GCSE; for example, mathematical treatments of radioactive decay and simple harmonic motion require calculus and, for the latter, trigonometry. Indeed, treatments that eschew this level of mathematics can often be more difficult to understand and are almost always less concise.

6. *How should post-16 maths provision change to meet those future requirements?*

u. One area that stands out as being a change from current A-level physics is the use of ideas and techniques from calculus. Calculus provides many opportunities for examining the world in the ways that are familiar to physicists – or those who apply physics.

v. Students should be familiar with the ideas of rates of change and accumulation and how they relate to physical situations and relationships between quantities (for example, acceleration is the rate of change of velocity and the total work done to stretch an elastic band is an accumulation of the work done during the extension).

w. The appendix provides some statements relating to calculus. It is important to note that it is not expected that students should be able to use advanced techniques to differentiate or integrate complicated functions. Instead, they should develop an understanding of the language of calculus and its meaning, a fluency in the basic techniques and an ability to apply them to a defined set of physical situations. They should be able to think with and set up simple mathematical models that relate to physical situations and involve simple differential equations.

7. *Is there a case for more (or all) students to study maths after the age of 16? To what level?*

x. The new Core Maths courses were introduced nationally in September 2015. However, these courses do not fulfil the requirements above: they require as many guided learning hours as an AS level and they are at too low a level for the needs of physics A-level.

y. Nevertheless, they highlight a slight paradox – that there is a post-16 course and qualification to support students who did not achieve one of the higher grades at GCSE and have dropped mathematical subjects altogether. Yet there is nothing for students who achieved an A* to B at GCSE and who are continuing with physics (or the sciences) and would benefit from developing their mathematical skills to a level higher than GCSE.

z. In the current system of free choice, it might be counter-productive to expect students to take an additional qualification alongside physics when they wouldn't have to in, say, economics. However, in a system in which it was compulsory for all students to study maths after the age of 16, it would be possible to design a bespoke course that supported physics (and other sciences) at A-level.

aa. There is also the question of who would teach any new mathematics content. There is already a shortage of maths teachers (as well as physics teachers) and the requirements for physics would entail having more than a basic competence in maths and maths teaching.

8. Please add any other comments or evidence you would like us to consider.

bb. Whilst physics students clearly benefit from taking an A-level in mathematics, there are questions of coherence and phasing of content. The mathematics A-level is spread over two years. Students will study calculus, differential equations and the differential of ex at a time that fits into the mathematics course rather than at a time that gives them the tools they need to learn about, for example, radioactive decay in a mathematical way. Thus, short of schools tying together the calendars of both mathematics and physics A-levels, it will be difficult for mathematics to perfectly complement the needs of those studying physics.

cc. One possibility would be a short course in mathematics that provides students with the tools to study physics in a mathematical way. This would not require the same timetable allocation as a full subject choice (for AS or A-level) and would allow them to continue to have complete choice over a full set of A-level subjects.

dd. Furthermore, it would enable schools to design the teaching of the mathematical content in a way that coheres with the physics course. For this reason, it might be the case that it would also appeal to students who are taking A-level maths; otherwise they would be at a disadvantage until they had covered the mathematics content in their A-level course.

ee. To satisfy the challenges of phasing and providing teachers, the new content could form part of the early topics in the AS level and A-level so that they could be co-taught. However, this would clearly need to be done without disrupting the pedagogical flow of the maths courses. This, we acknowledge, would be a real challenge.

ff. If such a course existed, it would require a qualification so as to reward the effort and time commitment of students. This qualification would not be the size of an AS level. It is worth emphasising that we are not advocating such a qualification in an environment of completely free choice (as we have now) because it would put physics in a unique and unattractive position.

gg. A more manageable and practical long-term solution would be a framework of baccalaureate-style qualifications at 18. An integrated portfolio would be the most effective way to ensure that students follow a coherent pathway through their post-16 education – one in which each subject supports the learning within the others. The precedent has been set by the Report of the Independent Panel on Technical Education[3] that has recommended 15 routes through technical education at level 3. The advantage of such frameworks are that:

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3 https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/536046/Report_of_the_Independent_Panel_on_Technical_Education.pdf

- The learning across all subjects (but in this case physics and mathematics) can be coherent and phased;
- It avoids learning the same thing twice (and differently) in different subject areas;
- It ensures that all students in a physics classroom have the same mathematical background (those not taking mathematics are not disadvantaged and left behind);
- It would allow more content to be covered because the whole would be more than the sum of the parts.

Appendix: Calculus statements

Physics A-level students should be able to

1. Interpret the derivative, dy/dx, of $y = f(x)$ as the gradient of the tangent to the graph of $y = f(x)$ at a point;

2. Differentiate simple functions of x including:
   $y = x^n$, ($n = -2$ to 4), sin(x), cos(x), e^x

3. Interpret dy/dt as the instantaneous value of the rate of change of y including:
   $v = ds/dt$, $a = dv/dt$, $P = dW/dt$, $I = dQ/dt$

4. Relate the area under a graph to the summation of the strips, each of area $f(x)\Delta x$.

   Interpretation of the integral, $\int y dx$, of $y=f(x)$ as the limit of the summation and that, for some functions, this provides a way of finding the new function that represents the area under the graph.

5. Interpret the definite integral, $z=\int y dt$, of $y=f(t)$ between two limits as the accumulation over time of a change in the quantity z including:

   $\Delta s=\int v dt$, $\Delta v=\int a dt$, $\Delta W=\int P dt$, $\Delta Q=\int I dt$

6. Integrate simple functions of x including:
   $y = x^n$, ($n = -2$ to 3 – including the special case of $n=-1$), sin(x), cos(x), e^x

7. Use integration to find the total work done between two points with a simple varying force.

8. Set up and solve first order differential equations from mathematical models of the form:
   $\Delta x = -kx.\ \Delta t$

9. Set up and solve the second order differential equation from mathematical models that take the form: $a = -\square^2 s$

For further information, please contact
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