A SPACE ODYSSEY

MOTION UNDER GRAVITY

PREAMBLE

This problem is adapted from an on-line knowledge enhancement module for a PGCE programme. It is used to cover the dynamics of rigid bodies and motion under gravity. The original material can be found at http://open.jorum.ac.uk/xmlui/handle/123456789/2969. This unit was run over one week with on-line support.

INTENDED LEARNING OUTCOMES

By the end of the module students should be able to:

- Define angular speed and angular acceleration
- Explain the difference between centrifugal force and centripetal acceleration
- Derive the properties of circular orbits under various forces
- Show knowledge of Newton's laws for rotational motion
- Define moment of inertia and explain its use
- Define and use Torque
- Apply conservation of angular momentum
- Distinguish between stable and unstable equilibrium
- Define simple harmonic motion

READING LIST

The reading list is that provided for the original module. Other equivalent textbooks are available.

READY TO STUDY

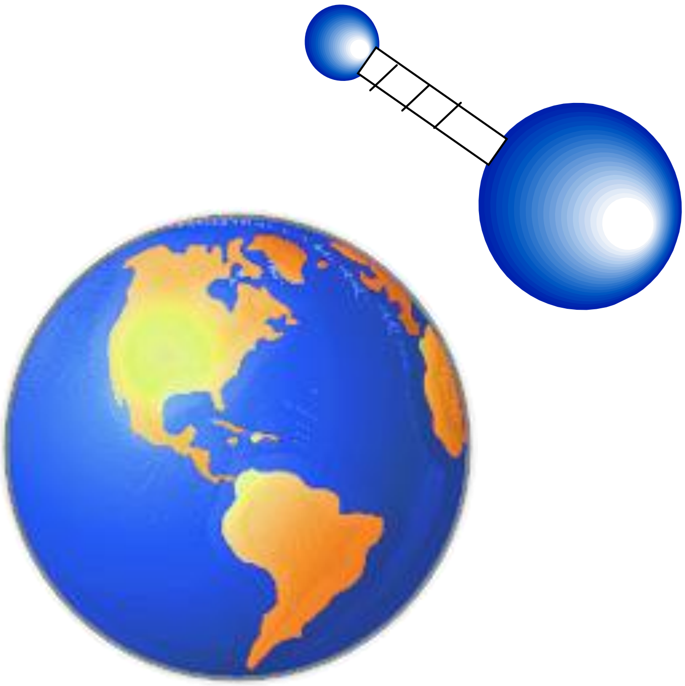
- Breithhaupt, J., ***Physics***. Palgrave Foundations.

ESSENTIAL

- Tipler, P.A., ***Physics for Scientists and Engineers***. Freeman.

PROBLEM STATEMENT

A design for a spaceship that would also function as an orbital space station might look like the dumbbell form of Spaceship *USS Discovery 1* from the film *2001: A Space Odyssey.* The picture shows an artist's impression with the spaceship moving round the Earth oriented like a plane flying through the air. Is there anything wrong with this?



The problem seems to involve the orientation of the spacecraft. Why would the orientation matter in space? But we're not in deep space in the picture: the spaceship is moving round the Earth, so there's Earth's gravity to consider. The spaceship is in fact an orbiting satellite. What can we say about the orbits of satellites round a parent body?

You will then need to look at the concept of stability. Here you'll see a difference between the behaviour of a point mass and an extended body in orbit. You will therefore have to consider how to model the mass distribution of the spaceship. Finally look at the motion of the spaceship about its equilibrium position. This will take you into the dynamics of rotational motion and the important topic of simple harmonic motion.

QUESTIONS FOR CLASS DISCUSSION

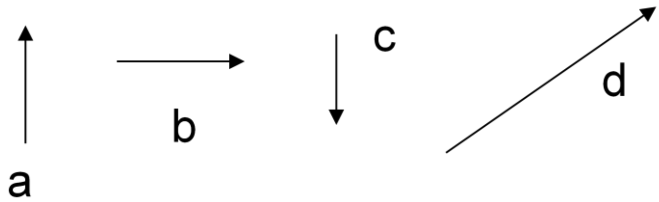
1. What is (i) the average angular speed (ii) the average acceleration of the Earth in its orbit about the Sun?
2. If the angular velocity is ω what how many orbits are completed per unit of time?
3. A satellite in orbit has (a) |PE| > KE (b) |PE| < KE or (c) |PE| = KE?
4. Put the following satellites in order of distance from Jupiter given their orbital periods in hours.

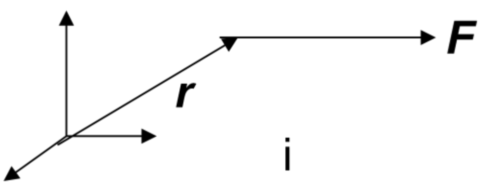
Mimas 22.62	Tethys 45.31	Titan 382.69
Enceladus 32.89	Rhea 108.42	Dione 65.69

5. The picture shows the Petronas Towers in Kuala Lumpur. The materials of the towers are clearly not in their lowest energy state. The building is obviously stable? Why?

The Petronas Towers. Photo (cc) monstermunch99

6. Applying the same logic to an airplane as to USS Discovery 1 would show that level flight is impossible and only a nosedive is stable. What is wrong with the argument in this case?
7. The actual (fictional) USS Discovery 1 had asymmetrical dumbells – what difference does this make to the stability argument?
8. A spinning ballet dancer draws in her arms thus halving her moment of inertia and doubling her angular velocity, so conserving angular momentum. Does her energy (a) increase, (b) decrease or (c) stay the same.
9. Match the couples (a, b, c or d) to the forces (i) and (ii):





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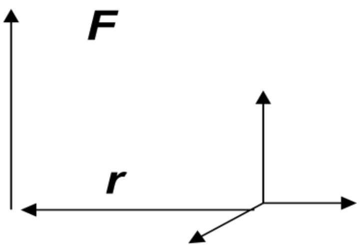


ii

10. Match the frequencies (i-iv) of SHM for a body of mass m near $x = 0$ to the potentials (a-c):

a) $U = m\omega^2 x^2$

b) $U = m\cos(\omega x)$ given that $\cos(\omega x)$ can be approximated as $1 - ½(\omega x)^2$ for small $\omega^2 x$

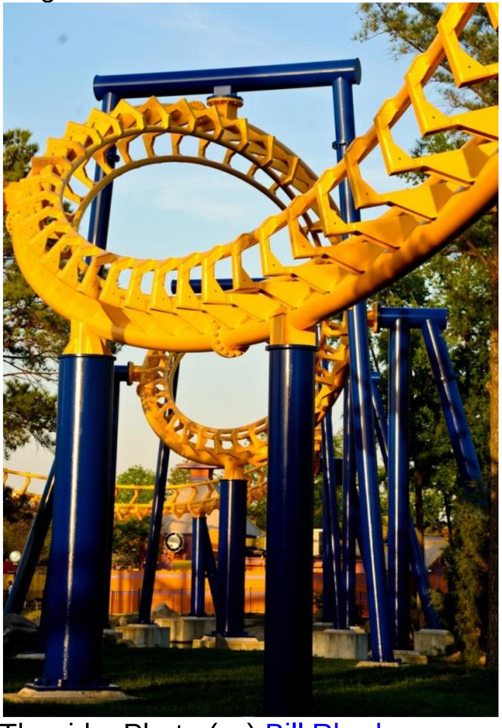
c) $\dfrac{m}{1-2\omega^2 x^2}$

(i) ω (ii) $\sqrt{2}\omega$ (iii) 2ω (iv) $\omega/2$

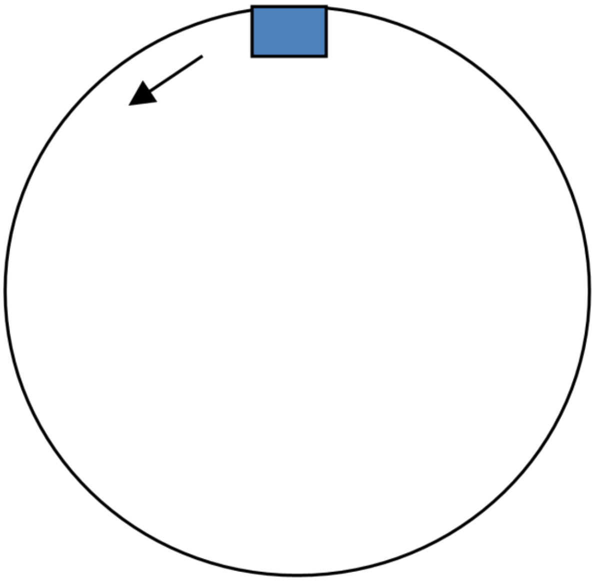
INDIVIDUAL EXERCISES

1. A planet in orbit is a bit like a pendulum (on the end of a gravitational 'string'). Kepler's third law of planetary motion gives the relation between the period of a planet in its orbit and the radius of the orbit. Suggest a form for such a relation from the formula for the period of the pendulum.

2. Assuming that the carts are not attached to the rails how big can a circular track in a fairground ride be?



The ride. Photo (cc) Bill Rhodes.



The Model

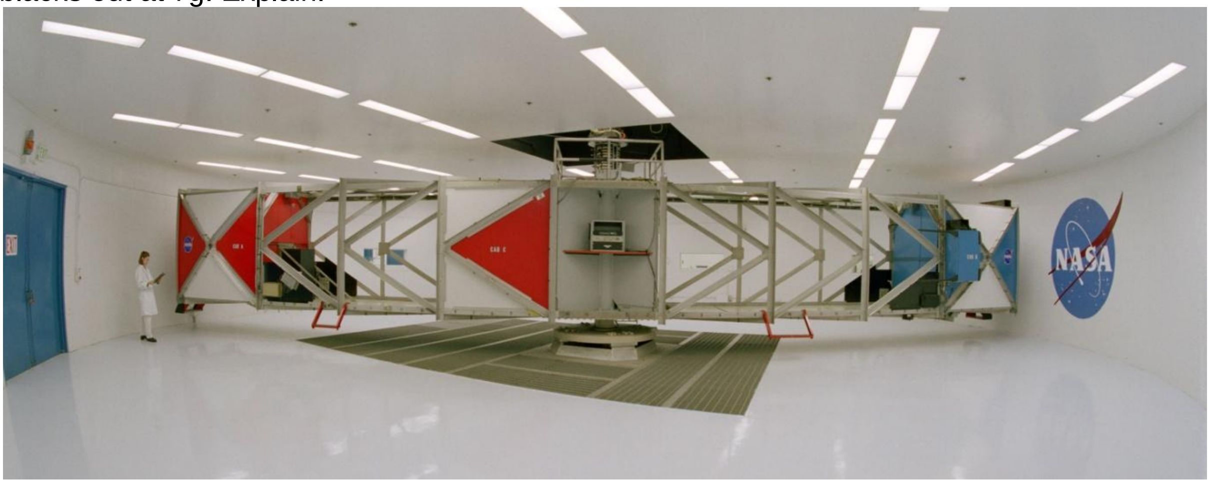
3. The Sun is losing mass constantly via the conversion of nuclear fuel and a solar wind. How does the loss of mass affect the orbit of the Earth? What impact does this have on climate change?

4. Galileo is credited with establishing that all bodies fall with the same acceleration under gravity. He did this not by dropping bodies from the leaning tower of Pisa, as legend has it, but by rolling balls down an inclined plane. This has the advantage of diluting gravity which makes it easier to measure the time of fall. However, Galileo was fortunate in the shapes of bodies he chose to compare. Does the shape of the rolling object make any difference? Would Galileo have made his discovery if he had compared the rolling motion of spheres and cylinders?

5. A Fairground rotor consists of a cylindrical room which can be spun about its axis of symmetry. Intrepid members of the public stand with their backs to the wall while the room is spun up, at which point the floor is removed. The people inside find themselves stuck to the wall. How fast must a 4m rotor be spinning before the floor can be lowered?

6. In order to remain in orbit a spacestation must be supplied with energy. An efficient way of doing this is to use the energy of visiting space shuttles as they undock, by paying out the shuttle on a long tether. How would this work?

7. In order to prepare for space missions astronauts are subject to high g-forces strapped into a centrifuge. A female of height (5'6") can withstand about 9g whereas a 6'3" male blacks out at 7g. Explain.



8. Where is the angular momentum in the Solar System? Don't forget you always have the option of transforming frames of reference. Try looking at it from the astronaut's point of view (at rest).

9. The Moon recedes from the Earth at the 3.8cm/year as a result of tidal torques exerted by the Earth on the Moon. How would you explain this? Estimate the change in the length of the year as a result of this.

10. Look up the maximum power and maximum torque and the corresponding revs (the angular speed of the engine) for your car (or for a car of your choice on the internet if you prefer). Work out the torque of your car at maximum power and compare this with the figure given for maximum torque.