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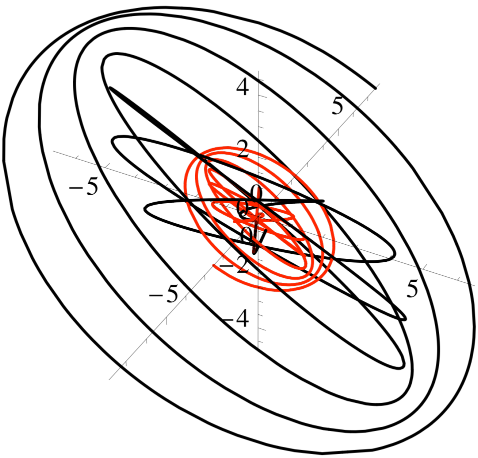

Motion of two black holes in a numerical simulation. The mass of the "red" black holes is three times that of the "black" one. The large black hole is spinning, and the precession of the orbital plane of the binary is clearly visible. Courtesy M. Hannam

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Welcome from the Chair

Another year, another newsletter...

For many of us 2010 has been a stressful year. The physic’s funding situation has not improved. In fact, the STFC cuts have now reached the point where some of the strongest groups in our area are seeing severe cuts. In addition, we have the government's austerity programme and drastic changes associated with the future hike in students fees. Not a pretty picture overall. I could go on, but it would only make me depressed. So... let's consider the positives.

As in previous years, the Gravitational Physics Group has been able to fund a range of interesting and relevant meetings in 2010. Our budget is small, but I think that the support that we give can make a real difference. However, in the last few months we have not had that many requests for support. Hopefully, this trend does not continue. We need to be active and keep a high profile in order to get through the difficult times. If you are planning a meeting relevant to gravitational physics, don't forget to ask for support from the GPG. We are also interested to hear your success stories, perhaps for the next issue of the newsletter?

This year's BritGrav meeting was a departure from the norm in that it was held in Dublin. I would like to thank the organisers for the initiative and for arranging a very successful event. Next year's meeting will also be different. BritGrav 2011 will form a part of the IOP Nuclear and Particle Physics Division (NPPD) conference in Glasgow (4-7 April 2011). Gravitational Physics part of this division, and when we were asked to take part we agreed (after some deliberations). There are pros and cons to this decision. On the positive side, we will be able to offer an exciting programme with very good invited speakers. Moreover, the draft programme has relevant talks in nuclear, particle and astro-particle physics. Of course, this is a big meeting, with associated costs etcetera. There is also a limited number of talks that we can accommodate in the planned parallel sessions. This is a clear change from the BritGrav philosophy. We certainly do not expect to stay with this format, but for a one-off it could be exciting and we hope to get good support from the community. In fact, I would encourage all students and postdocs to submit an abstract and help us demonstrate the strength of UK gravity, see

http://nppd.iopconfs.org/

for further details (note that the abstract submission deadline is 21 January 2011, quite soon!).

Whishing you a relaxing Holiday Season;

Nils

Events

BritGrav 10, Dublin, April 6th -7th 2010
Contributed by Brian Nolan

BritGrav headed west to Dublin this year, leaving the UK for the first time. The meeting was jointly organised by Dublin City University (Marc Casals and Brien Nolan) and University College Dublin (Adrian Ottewill), and took place in The Helix Arts Centre on the DCU campus. There were 53 participants in total, a little down on previous years. There were 29 presentations in all, with 15 of these being made by students. Participants came mainly from Ireland and the UK, and were joined by colleagues from Australia, Belgium, Germany, Russia, Spain, Sweden and the USA.

As usual, the talks ranged over a wide spectrum of subjects from gravitational physics and related areas. Topics included the self force, neutron stars, relativistic hydrodynamics, properties of spacetimes with symmetries, quantum issues in gravitation, cosmological perturbations and primordial black holes, hairy black holes, cosmic censorship, data analysis for gravitational wave detection and experimental tests of Newton's law. This last topic formed the subject matter of a talk by Emanuele Rocco of the Harvard-Smithsonian Centre for Astrophysics. This marked the beginning of a new feature of the BritGrav meetings, whereby the winner of the IOP Gravitational Physics Group Best Thesis Prize is invited to present an extended talk on their work. The presentations from the meeting are available electronically at \url{www.dcu.ie/~nolanb/BGX_talks.htm}

As in previous years, the IOP journal Classical and Quantum Gravity provided welcome sponsorship for the meeting, funding a reception on the first evening of the meeting. (Following a well-established tradition, participants then undertook a cultural tour of the host city, visiting various landmark buildings of Dublin which, coincidentally, were all licensed premises.) CGQ also sponsored the Best Student Talk award. This year - and for the first time - the panel of judges consisting of Carsten Gundlach, Adrian Ottewill and Elizabeth Winstanley found it impossible to decide between the top two talks, and a joint award was made to Emily Duffy of Dublin City University who spoke on ``Cauchy Horizon Stability in Self-Similar LTB Spacetimes" and Mark Fisher of Monash University who spoke on ``Solutions of the Einstein-Yang-Mills Equations". In presenting the award, Prof.\ Winstanley commented on the continued increase in the quality of the presentations at successive meetings.

The organisers would like to thanks the sponsors of the meeting: Classical and Quantum Gravity, the IOP Gravitational Physics Group, who provided partial funding for students attending the conference, DCU's Office of the Vice-President for Research, the Irish Mathematical Society and the School of Mathematical Sciences, DCU. Following last year's meeting in Cardiff and this year's in Dublin, BritGrav is scheduled to complete its circuit of the Celtic nations with a visit to Glasgow next year.

Current and future challenges in gravitational physics 14 May 2010, Institute of Physics, 76 Portland Place, London

Contributed by David Burton (d.burton@lancaster.ac.uk)

This popular meeting was organized by the IOP Gravitational Physics Group and was attended by approximately 70 participants from around the UK. The purpose of the meeting was to address the current status of gravitational physics research, and how the subject is expected to develop during the next decade.

The meeting began with an overview by Bernard Schutz (Max Planck Institute for Gravitational Physics [Albert Einstein Institute] and Cardiff) on "Gravitational Physics in the coming 15 years". The speaker explained how advances in gravitational physics will contribute to astronomy and particle physics. In particular, he explained that the next 15 years is likely to include the first direction detection of gravitational waves (GWs) by LSC and VIRGO and exploitation of GW astronomy; measurement of the cosmic GW background in the CMB; direct detection of a nHz astrophysical GW background by pulsar timings; the launch of LISA and exploration of the massive black hole population; the detection of the Higgs particle at LHC; higher precision cosmology (measurement of w to less than 1 part in one hundred); better population studies of intermediate-mass black holes; significant advances in understanding non-linear dynamics in General Relativity. In addition, he described a number of items that we might uncover : direct detection of a dark matter particle; evidence for structure in dark matter; evidence for cosmic strings; non-standard GW backgrounds; consequences of quantum gravity and string theory; low energy consequences of GUTs or string theory. His talk included a description of the GW community's plans until 2025 and the details of the operation of GW detectors, in particular LISA, and black hole merger simulations.

The second talk, entitled "Current and future challenges in gravitational wave observation", was given by Stephen Fairhurst (Cardiff) and focussed on ground-based detectors. He described the development of analytical waveforms to analyse GW emission from black hole binary inspiral, merger and ringdown and the efforts that are underway to develop templates that include spin. He explained that GWs provide complementary information to EM/neutrino data, and that GWs would be a precursor to the EM signal from many sources of interest. Accurate knowledge of the position of localized sources of GWs would be extremely useful and he noted that this could be achieved by an additional gravitational wave interferometer in Australia (AIGO).

The third talk, "Gravity and hydrodynamics" by Mukund Rangamani (Durham), was a summary of the relationship between black holes and fluid dynamics provided by the AdS/CFT correspondence. He discussed how black hole solutions can encode relativistic fluid configurations, and how they may be used to determine the transport properties of strongly coupled quantum field theories and their role in Quark Gluon Plasma simulations. He also commented on the possibility of using hydrodynamics to study the stability of black holes and the desire to use black holes
to understand fluid turbulence.

The final talk, "Challenges for gravitational physics : cosmological aspects" by Kazuya Koyama (Portsmouth), focussed on our lack of understanding of the nature of dark matter and dark energy. He described alternatives to General Relativity (such as Bekenstein's TeVeS theory and Horava-Lifshitz gravity) that solve the problem without having to introduce new matter, but explained that such approaches do have a number of difficulties. He finished by noting that future experiments should allow us to determine whether dark matter and energy are non-gravitational or gravitational in origin.

Contributed articles

Heat conduction in relativistic systems: alternatives and perspectives

Contributed by César López-Monsalvo, School of Mathematics, University of Southampton (cslm1x07@soton.ac.uk)

Does a moving body appear cold? This remarkably simple question, raised by the late Professor P. T. Landsberg some forty years ago, highlights a profound missing link in our current understanding of classical Thermodynamics and Relativity. Some might interpret this as the lack of a relativistic transformation law for temperature measure- ments (see [1, 2, 3]). There is, however, another view independent of any possible answer. It shifts the attention from a mere exercise in velocity transformations to an argument about thermal equilibrium between moving bodies and the inertial properties of heat [4, 5, 6, 7]. Such view promotes the discussion about the missing Lorentz transformation for temperature to the more elaborate —and physically meaningful— problem of heat exchange in a relativistic setting. Here, we present the essential structure of our recent efforts to tackle such problem from a variational approach [8].

Let us begin our discussion of heat from a non-relativistic point of view, where the notion of causality plays a less fundamental role. The heat equation is obtained by assuming the validity of the first and second laws of thermodynamics together with Fourier's law relating heat with temperature gradients in an un-relaxed manner. This implies that when two bodies at different temperatures are put in thermal contact, heat spontaneously flows from the warmer to the colder without any delay. Such conclusion cannot be satisfactory. One would expect the speed of thermal disturbances to be bound by the internal structure of the media they travel on. Cattaneo saw a way around this problem by introducing in a reasonable (but arbitrary) manner —a modification to Fourier's law which takes into account the characteristic time a material takes to react to thermal stimuli [9]

$$\mathbf{q} = -\kappa \nabla T \quad \rightarrow \quad \tau \dot{\mathbf{q}} + \mathbf{q} = -\kappa \nabla T. \tag{1}$$

Here q denotes the heat flux, T the temperature, κ the thermal conductivity, τ represents the relaxation time of the medium and the dot denotes time differentiation. Such amendment leads to a telegrapher equation for the propagation of heat signals, with a finite cap on the speed they can reach. It is worth mentioning that Fourier's law is the simplest, but not the most general, possible Ansatz to ensure that the second law is satisfied. This is done by explicitly making the change in entropy a quadratic function of the heat flow. We should keep this in mind in the forthcoming discussion.

In the relativistic case, the unbounded speed of thermal disturbances implied by the parabolic nature of the heat equation is more than mere inconvenience, it is indeed a fundamental problem. To give context to our discussion, let us consider the case where matter's motion is represented by a fluid whose normalised four velocity is given by u^a. The first attempt of a relativistic extension for the heat equation was due to Eckart [10]. The kind of model he proposed has become the stereotypical of a class of theories referred as first order. In this class, the entropy current of the model, denoted by a vector field s_a which is generally not aligned with the matter four-velocity, may only depend on terms which are linear in deviations from equilibrium, namely the heat flow or the shear viscosity. For this class of theories, the simplest way to impose the second law, which locally takes the form $s^a{}_{;a} \geq 0$, leads to a relativistic version of Fourier's law

$$q^a = -\kappa h^{ab} \left[T_{;b} + T \dot{u}_b \right], \tag{2}$$

where q^a represents the heat flux h^{ab} is a projector orthogonal to the matter flow and the semi-colon and dot denote covariant and proper time differentiation respectively. Being essentially identical to the first of the equations in (1), equation (2) inevitably produces a non-causal theory. Furthermore, as shown by Hisckock and Lindblom [11], it also suffers from stability problems. However, Eckart's proposal exhibits an extra piece of information which is missing in the non-relativistic treatment; the acceleration term. This purely relativistic effect can naturally be interpreted as being due to an effective "mass" per unit entropy given precisely by the temperature (see discussions in [5, 6, 8, 12]). It is worth mentioning that, recently, this term has been suggested to be the origin of the afore mentioned instabilities (see [13, 14]). However, we adopt here the vision that the appearance of the 4-acceleration in (2) is an inevitable, and physcally meaningful, feature of any relativistic theory of dissipation.

The failure of first order theories to produce a theory of heat conduction compatible with the principle of causality can be tracked down to their definition of the entropy current. In an effort driven by simplicity, it is not permissible to prematurely drop higher order terms in deviations from equilibrium, since they may give rise to linear terms after the differentiation required by the second law. This point of view is at the heart of the class of second order theories of heat conduction whose key contribution is the widely known Israel & Stewart model. Here, the entropy current used by Eckart is extended to include all the possible second order combinations of dissipative effects [15, 16, 17]. This strategy, analogous to the Grad's 14-moment theory, is firmly ground on kinetic theory and provides a causal and stable account of relativistic dissipation. It is not our intention here to further explain this particular theory, but to note that, in spite of its success on stability and causality tests, the price to pay is the introduction of a set of second order couplings that, in principle, can be measured but which cannot be obtained within the realm of the theory.

Owing to an increasing interest found in applications of high-energy relativistic plasmas, attention to relativistic theories of dissipation of the Israel & Stewart type has been regained. In many such applications, whose main source of dissipation is due to viscosity, heat can effectively be relegated as a secondary effect. Indeed, it was these kind scenarios, where the ratio of viscous to thermal resistivity can be pushed down to order unity, the main source of motivation for the development of Israel & Stewart second order theory. Our interest, however, follows from quite a different motivation. It ranges from recent efforts to model the dynamics of super-fluid neutron stars [12, 18] to high-energy gases with photons providing the dominant pressure contribution [6].

In this brief account of a long-standing problem in Relativity, it is our wish to shed some light into a fundamentally more satisfactory variational approach to relativistic heat conduction [6, 7, 19]. Historically, due to an over simplified view of the components of the theory, this model, pioneered by Carter, failed the tests of stability and causality. However, it was later shown by Priou [20] that the predictions of the complete Carter's theory are equivalent, and at second order physically indistinguishable, to those of Israel & Stewart. Despite Priou's efforts, the issue has been closed in favour of the later.

Perhaps the most attractive feature of a variational construction of relativistic heat conduction is that, once the equation of state (or, equivalently, the Lagrangian density) of the system is known, the theory contains no free parameters. The price for this, however, is the inclusion of transport quantities in the "Lagrangian" density of the system. This is, nevertheless, inevitable in any circumstance where one's aspiration is to describe, at least to linear accuracy, situations departing from local thermal equilibrium.

In Carter's variational approach to heat conduction, one considers a multi-fluid system whose species are represented by a particle number density current n^a and an entropy flux s^a. These two currents, together with the spacetime metric, constitute the fundamental fields of the matter sector for the Einstein-Hilbert action. General covariance requires the Lagrangian to be a proper scalar, therefore, it should depend only on covariant combinations of its fundamental fields. If we consider the metric as a passive field, the Lagrangian density can only depend on combinations of the two fluxes, which includes the relative flow

between them. This is precisely what we meant by the inclusion of transport quantities in the Lagrangian density.
A constrained variation of the matter action, whose Lagrangian density has the characteristics described in the preceding paragraph and whose constrain is the one imposed by the conservation law of the particle number density flux, allows us to write the local conservation of energy and momentum as a “force balance” equation. It is worth noticing that each of the individual “forces” appearing in such balance, takes a form completely analogous to the Lorentz force in a general relativistic setting (see section 2 in [8]). The requirement for the local energy conservation law to follow as a Noether identity of the variational principle, only needs one of the currents to be strictly conserved, $n^{a}{}_{;a} = 0$ say. This allows an “extra” freedom to allocate the second law of thermodynamics by the non-vanishing of the production term, $s^{a}{}_{;a} \geq 0$.

One of the central problems to be faced before giving a real thermodynamic interpretation to the variational construction, lies in the correct interpretation of temperature measurements. As stressed in the opening sentence, this is a non-trivial problem in relativity and, therefore, a choice of frame is forced upon us. In this simple case, where we have only one conserved current, the choice is somewhat natural. In the spirit of physical interpretation, we can chose to equip each infinitesimal bit of the matter fluid with a thermometer and consider all physical measurements to be taken on, and with respect to, the matter frame. In such case, one can show that, indeed, the projection of the momentum canonically conjugate to the entropy flux into the matter frame, which we denote here by $\theta\|$, corresponds to the thermodynamic temperature in the sense of Gibbs[1]. Having clarified this, it is not difficult to show that the heat flux relative to the matter frame takes the form [8]

$$\check{\tau}\left[\dot{q}^{a} + u^{c;a} q_{c}\right] + q^{a} = -\check{\kappa} h^{ab}\left[\theta^{\|}_{;b} + \theta^{\|}\dot{u}_{b}\right], \tag{3}$$

where all the quantities, with the exception of $\check{\tau}$ and $\check{\kappa}$ which correspond to the effective relaxation time and thermal conductivity, have been properly introduced in the text. This result, which is a relativistic generalization of Cattaneo’s equation [the modified Fourier’s law in (1)], follows directly from the variational principle together with the simplest assumption (in the sense previously discussed) to make the entropy production satisfy the second law of thermodynamics.

The structure of (3) combines all the features present in both (1) and (2), including the acceleration term which in the variational context arises as a consequence of

[1] The rate of change of energy with respect to entropy with all other independent thermodynamic uantities fixed.

the equations of motion. The “check” marks indicate that these quantities depend on higher than second order deviations from thermal equilibrium and, therefore, a comparison with an analogous expression obtained from the Israel & Stewart theory is beyond the scope of present means of verification.

Let us emphasize that the main objection to Carter’s approach was based on issues about stability and causality. This is a consequence of the simplistic spirit of the original model, ignoring a crucial effect present in almost every multi-fluid system; entrainment. In addition to Priou’s argument on the equivalence up to second order of Carter and Israel & Stewart theories, using a recent generalization of the two-stream instability analysis [21], it is possible to asses the stability and causality of a proposed dissipative theory at the level of the equation of state or Lagrangian density.

We have presented here a summary of the most significant efforts attempted to obtain a physically sound theory of heat conduction compatible with the principle of local causality. In spite of the recent stir made on first order theories, in our view, physical indicators such as the presence of second sound in materials hint that the correct physics to model departures from thermal equilibrium cannot be less than second order. Although the Israel & Stewart model is the most prominent and widely used tool to describe dissipative systems, the additional couplings —necessarily introduced in their expansion— give the theory an effective, rather than fundamental, character. In this sense, the variational formalism not only provides us with an alternative to the Israel & Stewart model, it centres the attention in the dynamical actors of any canonical theory; the canonical conjugate momenta. This observation makes the multi-fluid construction a very powerful tool which may help us tackle deeper problems on non-equilibrium thermodynamics of relativistic systems.

Acknowledgements

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References

[1] P. T. Landsberg, *Does a monving body appear cool*?, Nature, 212

[2] P. T. Landsberg and G. E. A. Matsas, *The imposibility of a universal relativistic temperature transformation*, Physica A Statistical Mechanics and its Applications, **340** 92-94, 2004

[3] T. K. Nakamura, *Lorentz transformation for black-body radiation temperature*, Europhys. Lett. **88**, 20004, 2009

[4] W. Israel, *Covariant fluid mechanics and thermodynamics; and introduction*, In Relativistic fluid dynamics, vol. 1385 (eds. A. Anile & Y. Choquet-Bruhat), Springer Lecture Notes in Mathematics, pp 153-210, 1989.

[5] J. Ehlers, *Survey of general relativity theory, In Relativity, Astrophysics and Cosmology*, vol. 38 (ed. W. Israel), Astrophysics and Space Science Library, D. Reidel Publishing Company, pp 1-125, 1973.
[6] B. Carter, *Covariant theory of conductivity in ideal fluid or solid media*, In Relativistic fluid dynamics, vol. 1385 (eds. A. Anile & Y. Choquet-Bruhat), Springer Lecture Notes in Mathematics, pp 1-64, 1989.
[7] B. Carter, *Regular and anomalous heat conduction: the canonical diffusion equation in relativistic thermodynamics*, Journées Relativistes (eds M. Cahen, R. Debever & J. Geheniau), Universiť e Libre de Bruxelles, pp 12-27, 1976.
[8] C. S. López-Monsalvo and N. Andersson, *Thermal dynamics in general relativity*, Proc. Roy. Soc. A Published online 1 September 2010, doi: 10.1098/rspa.2010.0308, 2010.
[9] C. Cattaneo, *Sulla conduzione del calore*, Atti Semin. Mat. Fis. Univ. Modena **3**, 83-101, 1948.
[10] C. Eckart, *The thermodynamics of irreversible processes. III. Relativistic theory of the simple fluid*, Phys. Rev. **58**, 919-924, 1940
[11] W. A. Hiscock and L. Lindblom, *Generic instabilities in first-order dissipative relativistic fluid theories* Phys. Rev. D **31**, 725-733, 1985
[12] N. Andersson and G. Comer, *Variational multi-fluid dynamics and causal heat conductivity*, Proc. Roy. Soc. A **466**, 1373-1371, 2010.
[13] A. L. García-Perciante, L. García-Colín and A. Sandoval-Villalbazo, *On the nature of the so-called generic instabilities in dissipative relativistic hydrodynamics*, Gen. Rel. Grav. **41**, 1645-1654, 2009.
[14] L. S. Garcia-Colín and A. Sandoval-Villalbazo, *Relativistic non-equilibrium thermodynamics revisited*, J. Non- Equilibrium Thermodyn **31**, 11-22, 2006.
[15] J. M. Stewart, *On transient relativistic thermodynamics and kinetic theory*, Proc. Roy. Soc. A **357**, 59-75, 1977
[16] W. Israel and J. M. Stewart, *On transient relativistic thermodynamics and kinetic theory II*, Proc. Roy. Soc. A **365**, 43-52, 1979
[17] W. Israel and J. M. Stewart, *Transient relativistic thermodynamics and kinetic theory*, Ann. Phys. **118**, 341-372, 1979
[18] N. Andersson, *Gravitational waves from instabilities in relativistic stars*, Class. Quant. Grav. **20** R105-R144, 2003.
[19] B. Carter, *Conductivity with causality in relativistic hydrodynamics: the regular solution to Eckarts problem*, In Highlights in gravitation and cosmology, (eds. B. R. Iyer, A. Kembhavi, J. V. Narlikar and C. V. Vishveshwara), Cambridge University Press, 1988.
[20] D. Priou, *Comparison between variational and traditional approaches to relativistic thermodynamics of dissipative fluids*, Phys. Rev. D **43** 1223, 1991.
[21] L. Samuelsson, C. S. López-Monsalvo, N. Andersson and G. Comer, *Relativistic two-stream instability*, Gen. Rel. Grav. 2010

Conformal Killing-Yano tensors: Symmetries of the Dirac equation in curved spacetime

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Ever since its discovery in 1928, the Dirac operator has played a central role in physics and more recently geometry. The Dirac equation governs the behaviour of spin 1/2 fermions such as electrons or protons; it is indispensable in the construction of the Standard Model of particle physics. Using the vielbein formalism the Dirac operator can be also studied in curved spacetime. Even more fundamentally, this operator is intimately tied to the geometrical structures. For example, a refined notion of geometry (especially suitable for noncommutative spaces) is obtained by treating the Dirac operator as fundamental and the metric as a derived object. In this brief overview we concentrate on symmetries of the Dirac operator in a fixed curved background.

In curved spacetime nontrivial (not associated with isometries) symmetry operators of the Dirac operator were for the first time studied by Carter and McLenaghan [1]. These authors constructed a commuting with Dirac operator in four spacetime dimensions cor- responding to a (hidden) symmetry described by a Killing–Yano 2-form. The existence of such symmetry imposes nontrivial restrictions on the background geometry. In particular, the spacetime must be algebraically special—of type D [2]. Remarkably, it turns out that this symmetry exists in the most general stationary (vacuum) black hole spacetime described by the Kerr geometry [3]. (In fact, with a few extra conditions this symme- try uniquely characterizes the Kerr solution among all vacuum solutions of the Einstein equations, see, e.g., recent paper [4] and references therein.) Consequently, in the Kerr geometry there exists a complete set of mutually commuting symmetry operators, one of which is the Dirac operator, which underlies the separability of the Dirac equation in this spacetime [5].

The paper [1] stimulated subsequent developments in the study of symmetry operators of the Dirac operator and separability of the Dirac equation in curved spacetime. In particular, the most general first-order operator commuting with the Dirac operator in 4D was constructed by McLenaghan and Spindel [6]. This work was later extended by Kamran and McLenaghan [7] to R-commuting symmetry operators, i.e., operators whose commutator is proportional to the Dirac operator itself. Such operators map solutions of the massless Dirac equation to solutions and correspond to symmetries which are conformal generalizations of Killing vectors and Killing–Yano tensors. An example of a spacetime which admits this (weaker) symmetry is C-metric and its rotating, charged, or cosmological constant generalizations described by the Plebanski–Demianski metric element. The massless Dirac equation separates in all these spacetimes.

The theory of separability of the Dirac equation in curved spacetime is far from being completely understood. This is contrary to the Hamilton–Jacobi and Klein–Gordon equations whose separability is well characterized; it is linked to the existence of Killing vectors and rank-2 Killing tensors forming the so called separability structure in which the prominent role play the (symmetric) Schauten–

Nijenhuis brackets [8, 9]. For the Dirac equation, on the other hand, it was only recently realized that its separability cannot be entirely characterized by the existence of the first order symmetry operators [10]; a counterexample of a space where the Dirac equation separates but the separability is underlied by an operator of the second order is known. This led people to study higher-order symmetry operators of the Dirac operator, e.g., [11]. It also means that the theory of separability of the Dirac equation must reach outside the realms of the factorizable systems [12].

With recent developments in string theory and various supergravities physicists have become interested in the Dirac equation in more general spacetimes—of 'arbitrary' dimen- sion and signature as well as spacetimes where the metric is supplemented by other matter fields (fluxes) which couple to the spinor and modify the Dirac equation. It is the aim of this letter to summarize recent progress in the study of symmetry operators and separability theory of the Dirac equation in these more general setups. We start with the beautiful and general result of Benn et. al. [13], valid in all dimensions n and arbitrary signature. The central role in this result play conformal Killing–Yano (CKY) p-forms, i.e., p-forms ω_p obeying [14]

$$\nabla_a(\omega_p)_{a_1\ldots a_p} = \frac{1}{p+1}(d\omega_p)_{aa_1\ldots a_p} + \frac{p}{n-p+1} g_{a[a_1}\nabla_{|b|}(\omega_p)^b{}_{a_2\ldots a_p]}\,. \tag{1}$$

Such p-forms are special—their covariant derivative splits into the exterior and divergence parts—they are in the kernel of a twistor operator $K_a\omega_p$= 0. If in addition the divergence part is zero we have Killing–Yano (KY) tensors, whereas we have closed conformal Killing– Yano (CCKY) tensors when $d(\omega_p) = 0$.

Theorem. The most general first-order operator S which R-commutes with the Dirac operator $D = \gamma^a\nabla_a$, i.e. obeying [S,D] = RD for some R, is given by $S = S_\omega + \alpha D$, where
$\alpha = \sum_p \frac{1}{p!}(\alpha_p)_{a_1\ldots a_p}\gamma^{a_1\ldots a_p}$
and α_p are arbitrary p-forms, and S_ω is defined in terms of an inhomogeneous CKY form
$\omega = \sum_p \omega_p,$
ω_p obeying (1), as

$$S_\omega = \sum_{p=0}^{n}\Big\{\gamma^{a_1\ldots a_{p-1}}(\omega_p)^a{}_{a_1\ldots a_{p-1}}\nabla_a + \frac{1}{2(p+1)^2}\gamma^{a_1\ldots a_{p+1}}(d\omega_p)_{a_1\ldots a_{p+1}} + \frac{(n-p)}{2(n-p+1)}\gamma^{a_1\ldots a_{p-1}}\nabla_b(\omega_p)^b{}_{a_1\ldots a_{p-1}}\Big\}\,. \tag{2}$$

Apart from the (inevitable) freedom of adding arbitrary form α this theorem states that in all dimensions and signatures the first order Dirac symmetry operators are in one-to-one correspondence with CKY symmetries. It generalizes the 4D results of Kamran and McLenaghan [7]. The result can be also viewed as an operator (or 'quantum') version of the corresponding statement on worldline supersymmetry of classical spinning particles [15].

In the presence of fluxes, instead of the 'bare' Dirac operator D, one considers the *modified Dirac* operator *D* of the form

$$\mathcal{D} = D + B = \gamma^a \nabla_a + \sum_p \frac{1}{p!} B_{a_1 \dots a_p} \gamma^{a_1 \dots a_p} \,. \tag{3}$$

This includes the case of a massive Dirac operator, the Dirac operator minimally or non- minimally coupled to a Maxwell field, the Dirac operator in the presence of torsion as well as more general operators. Symmetry operators of this operator were studied in [16, 17, 18]. These operators correspond to a generalized symmetry, described by inhomogeneous form ω, which is a solution to the generalized conformal Killing–Yano system—a flux generalization of the twistor equation (1). Unlike (1), however, this system couples in general various p-form parts of ω. Moreover, except trivial fluxes, solutions of this system are subject to additional (differential) constraints [18].

Of special interest is the case when the Dirac spinor couples to a skew-symmetric torsion and a U(1) field

$$\mathcal{D} = \gamma^a \nabla_a + i\gamma^a A_a - \frac{1}{24}\gamma^{abc} T_{abc} \,. \tag{4}$$

In that case the generalized conformal Killing–Yano system decouples into a system of independent equations of the form (1) for homogeneous parts ωp where ∇ = ∇T is now a connection with torsion T. In other words, the study of symmetries of operator (4) leads uniquely to conformal Killing–Yano equations in the presence of torsion, KaT ωp = 0, introduced in [19].

Among all possible symmetry operators *commuting operators* play a prominent role —their eigenvalues yield the 'quantum numbers' which characterize the solution, the constants of motion. By specifying α in the previous theorem, we have the following [20]:

Corollary. The most general first-order operator S which commutes with the Dirac oper- ator D splits into the Clifford even and Clifford odd parts, $S = S_e + S_o$. The odd part S_o is in one-to-one correspondence with CCKY even-forms, the even part S_e is given by KY odd-forms.

Concrete form of these operators can be found in [20]. This corollary can be straightforwardly extended in the presence of U(1) and torsion fluxes.

Separability of the (modified) Dirac equation has been recently studied in various higher-dimensional black hole spacetimes and spacetimes with non-trivial fluxes. In par- ticular, the massive Dirac equation was proved separable in the most general known Kerr-NUT-(A)dS spacetime in all dimensions [21]. Similar to 4D, this result can be justified by the existence of a complete set of mutually commuting first order operators [20]. Such oper- ators are given in terms of Killing vectors and CCKY even-forms which can be genererated from the principal conformal Killing–Yano tensor present in the spacetime [22]. Slightly more generally, separability of the torsion modified Dirac equation was demonstrated in the presence of U(1) and torsion fluxes of the Kerr–Sen geometry and its higher-dimensional generalizations [23] as well as in the most general spherical black hole spacetime of minimal gauged supergravity [24]. In all cases such separability can be justified by the existence of a generalized Killing–Yano tensor with torsion and the corresponding complete set of first order symmetry operators.

Despite the great progress in the study of symmetries of the (modified) Dirac operator in curved spacetime there still remain many fundamental open questions. The theory is well established for the first-order symmetry operators—they are essentially in one-to- one correspondence with (conformal) Killing–Yano tensors and their flux generalizations. Such operators stand behind separability of the (modified) Dirac equation in ‘physical backgrounds’ such as the most general Kerr-NUT-AdS spacetime in all dimensions, the black hole background of minimal gauged supergravity, or the Kerr-Sen geometry of string theory. In general, however, the first-order operators are not enough to fully characterize all possible symmetries and all separable systems; one has to consider higher-order symmetry operators. Why are all known ‘physical examples’ of separable systems characterized by first-order symmetry operators? Can we find all higher-order symmetry operators? Do they form some kind of algebra? Is there a separability theory for the Dirac equation, similar to the ‘separability structure’ of the Hamilton–Jacobi equation? These are interesting open questions left for future studies.

References

[1] B. Carter and R. G. McLenaghan, *Generalized total angular momentum operator for the Dirac equation in curved space-time*, Phys. Rev. D**19** (1979) 1093–1097.
[2] C. D. Collinson, *The existence of Killing tensors in empty space-times*, Tensor **28** (1974) 173.
[3] R. Penrose, *Naked singularities*, Annals N. Y. Acad. Sci. **224** (1973) 125–134.
[4] T. Backdahl and J. Valiente Kroon, *Geometric Invariant Measuring the Deviation from Kerr Data*, Phys. Rev. Lett. **104** (2010) 231102, [arXiv:1001.4492].
[5] S. Chandrasekhar, *The Solution of Dirac’s Equation in Kerr Geometry*, Proc. Roy. Soc. Lond. A**349** (1976) 571–575.

[6] R. McLenaghan and P. Spindel, *Quantum numbers for dirac spinor fields on a curved space-time*, Phys. Rev. D **20** (1979) 409.
[7] N. Kamran and R. G. McLenaghan, *Symmetry operators for neutrino and Dirac fields on curved space-time*, Phys. Rev. D**30** (1984) 357–362.
[8] S. Benenti and M. Francaviglia, *Remarks on certain separability structures and their applications to general relativity*, Gen. Rel. Grav. **10** (1979) 79–92.
[9] E. G. Kalnins and W. Miller, Jr, *Killing tensors and nonorthogonal variable separation for Hamilton–Jacobi equations*, SIAM J. Math. Anal. **12** (1981) 617.
[10] M. Fels and N. Kamran, *Non-factorizable separable systems and higher-order symmetries of the Dirac operator*.
[11] R. G. McLenaghan, S. N. Smith, and D. M. Walker, *Symmetry operators for spin-1/2 relativistic wave equations on curved space-time*.
[12] W. Miller, Jr, *Mechanisms for variable separation in partial differential equations and their relationship to group theory*, in Symmetries and nonlinear phenomena, (Singapore), pp. 188–221, World Scientific, 1988.
[13] I. M. Benn and P. Charlton, *Dirac symmetry operators from conformal Killing-Yano tensors*, Class. Quantum Grav. **14** (1997) 1037–1042, [gr-qc/9612011]; I. M. Benn and J. Kress, *First-rder Dirac symmetry operators*, Class. Quantum Grav. **21** (2004) 427.
[14] S. Tachibana, *On conformal Killing tensor in a riemannian space*, Tohoku Math. J. 21 (1969) 56; T. Kashiwada, *On conformal Killing tensors*, Nat. Sci. Rep. Ochanomizu Univ. **19** (1968) 67–74.
[15] G. W. Gibbons, R. H. Rietdijk, and J. W. van Holten, *SUSY in the sky*, Nucl. Phys. B**404** (1993) 42–64, [hep-th/9303112].
[16] O. Acik, U. Ertem, M. Onder, and A. Vercin, *First-order symmetries of Dirac equation in curved background: a unified dynamical symmetry condition*, arXiv:0806.1328.

How will we really use numerical relativity to detect gravitational waves?

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It was big news when the "binary black hole problem" was solved in 2005 [1,2,3]. For the first time Einstein's equations could be solved on a computer for a system of two black holes as they orbited each other a few times, fell together and merged and released a phenomenal burst of gravitational radiation. Previous simulations ran for no more than one orbit (and usually much much less), then became unstable and produced nonsense. The problem of making them work had kept researchers and their supercomputers occupied for more than 40 years.
Now, not only was a hard problem finally solved, but perhaps numerical relativity (NR) could contribute to a much larger scientific effort: the first direct detection of gravitational waves (GWs). The idea went like this. Gravity is weak and GWs are incredibly difficult to detect. An international network of detectors (LIGO, Geo, Virgo, and Tama) was in operation, but the chance that a signal would be clearly visible above background noise was small. The best hope of uncovering a signal from the noise was to first know exactly what it looked like, and to match the

detector data against a bank of these theoretical signals. The strongest imaginable source of GWs would be the merger of two black holes, and so the best way to make the first GW detection would be to have in hand the general relativistic prediction of the waves from a merger – and the only way to calculate that was from numerical simulations.

That was how many numerical relativists saw it, myself included. But this was a misleadingly simplistic picture. The role of NR simulations in producing banks of theoretical templates for GW searches has turned out to be quite different.

To begin with, NR waveforms cannot be just "used" in a GW search. In a matched filter search, a template bank of thousands of waveforms is required, densely populating the parameter space of binary configurations, which is made up of the masses of the two black holes, and their spins, and the location and orientation of the binary in the sky, and its distance from the detector. The waveform from a single numerical simulation can be converted into multiple GW signals that span many of these parameters; we can freely alter the binary's sky location, orientation and distance, and we can even rescale the waveform to change the total mass of the binary. But we still have waveforms for only one value of the ratio of the black-hole masses, and one value of their spins.

In addition, numerical waveforms include only the last few black-hole orbits before merger. The longest simulations to date include about fifteen orbits [4]. If a binary has a total mass more than fifty times that of the sun, then the fifteen-orbit simulation will produce the entire waveform that the initial LIGO detectors would be sensitive to. But the detectors will be sensitive to a much *longer* waveform if the binary's mass is lower. Astrophysical models predict that most potential signals will come from binaries of around 10 solar masses, and for these we need theoretical waveforms that include *hundreds* of orbits before merger. The numerical waveforms are too short.

So: we cannot produce a template bank out of one numerical waveform, and even if we could, it would only be useful in searches for binaries about five times more massive than the ones we expect to see.

And for low-mass binaries, we already have theoretical templates, based on the post-Newtonian (PN) approximation to general relativity. PN theory becomes less accurate as the binary approaches merger, but it should be accurate enough to allow detections of a 10-solar-mass binary.

When viewed this way, it doesn't sound like NR simulations are very useful for detection of GWs at all. So, what do we need NR simulations for? Why are we still keeping warm supercomputer centres all over the world?

For a start, the accuracy of the PN waveforms is difficult to determine. The PN expansion does not have clear convergence properties, and therefore error estimates in PN waveforms cannot be rigorously calculated. It was only with the calculation of the fully general relativistic waveforms from numerical simulations that the accuracy of the PN waveforms could begin to be properly quantified. We could search for GWs with PN templates, but we couldn't be sure how well they would work until we had NR waveforms to compare with.

What we've also found is that, while PN templates are acceptable for detection of binaries up to 10 solar masses, full inspiral-merger-ringdown (IMR) waveforms will be needed for more massive binaries [5]. These are not expected to be common, but our astrophysical expectations carry large uncertainties; the collected estimates of detection rates even for neutron-star binaries, which are much better understood, vary by two orders of magnitude [6]. And just *one* signal from a 50-solar-mass binary would provide us with a merger and ringdown signal in the heart of the detector's sensitivity band, our first observation of the strong-field regime of general relativity. Beyond detection (which should also be possible with burst searches), IMR waveforms will be essential for extracting physical information from the observation – the black-hole masses, their spins, the location of the binary in the sky, and the mass and spin of the final merged black hole.

This is really the key application of NR simulations: "parameter estimation", or, more simply, reading the physics from the observations. Even if IMR waveforms aren't necessary for the first detection, they will be crucial for GW astronomy.

What about the problem that NR waveforms are too short, and that they only cover discrete points in the parameter space? The length problem can be overcome by stitching together PN and NR waveforms. Then, given a series of hybrid PN+NR waveforms, it's possible to produce an analytic phenomenological model of generic waveforms, with free coefficients in the model determined by the hybrid waveforms. In other words, we make a curve fit through parameter space. The resulting phenomenological model can be used to generate template banks across the parameter space. So far this has been done for the "simpler" corners of parameter space, first for nonspinning binaries, and then for binaries with non-precessing spins [7]. A related technique is to use NR simulations to calibrate free parameters in an effective-one-body (EOB) model [8].
There is still a long way to go. Most astrophysical black holes will be spinning, and the spins will precess during the inspiral. Sufficiently sampling the spinning-binary parameter space in order to produce an accurate analytic model will be a huge computational and modelling challenge, although an international numerical- and analytical-relativity (NRAR) effort is already underway to meet it [9]. A second large-scale collaboration, the NINJA project, is working to understand how current search methods perform when there are full IMR waveforms in the data, which were of course unavailable for tests before the solution of the binary black hole

problem [9]. And finally, we still don't really know how to perform an efficient search of the data for generic spinning binaries, even if the theoretical waveforms were available.

These are the issues that numerical relativists, analytic relativists, and data analysts are now tackling. These problems weren't clear before NR simulations became possible, and some of them aren't yet fully clear. But the challenge now is to understand and solve them in the few years remaining before the first detections and the beginning of GW astronomy.

References

[1] F. Pretorius, Phys. Rev. Lett. 95, 121101 (2005.
[2] M. Campanelli, C. O. Lousto, P. Marronetti, and Y. Zlochower, Phys. Rev. Lett. 96, 111101 (2006).
[3] J. G. Baker, J. Centrella, D.-I. Choi, M. Koppitz, and J. van Meter, Phys. Rev. Lett. 96, 111102 (2006).
[4] M. Boyle, *et. al.*, Phys. Rev. D 76, 124038 (2007).
[5] A. Buonanno, *et. al.*, Phys. Rev. D 80, 084043 (2009).
[6] J. Adabie, *et. al.*, Class. Quant. Grav., 27, 173001 (2010).
[7] P. Ajith, *et. al.*, Phys. Rev. D 77, 104017 (2008).
[8] T. Damour and A. Buonanno, arXiv:0906.1769.
[9] www.ninja-project.org.

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